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Formulation Requirements

Differential Correction Module (DCM)

Mission Planning and Analysis Division
September 1979



National Aeronautics and
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Lyndon B. Johnson Space Center
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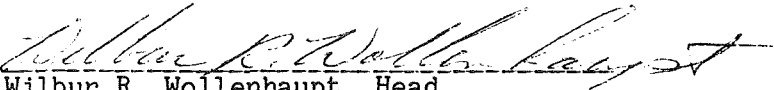
LEVEL C
ORBIT DETERMINATION PROCESSING

FORMULATION REQUIREMENTS


DIFFERENTIAL CORRECTION MODULE (DCM)

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PREFACE

The Mathematical Physics Branch/Mission Planning and Analysis Division has the responsibility to provide the functional ground navigation software formulation requirements for the Mission Control Center (MCC) low-speed-processing phases during Operations Project Shuttle (OPS).

The ground navigation software formulation requirements are logically organized into volumes. This organization is presented in the accompanying table. The material in each volume presents the level C formulation requirements of the processors and modules required to process low-speed-tracking data and perform orbit determination and other related navigation computations. Each volume describes the formulation requirements of the identified processor or module specified in the OPS MCC Ground Navigation Program Level B Software document (ref. 1). The inputs and outputs required to accomplish the functions described are specified. Flow charts defining the sequence of mathematical operations and display and control processing required to satisfy the described functions are included in the document where appropriate.

OPS MCC GROUND NAVIGATION PROGRAM LEVEL C SOFTWARE REQUIREMENTS

ORBIT DETERMINATION PROCESSING FORMULATION DOCUMENT

Volume I	Introduction and Overview
Volume II	Low-Speed Input Processor (LSIP)
Volume III	Bias Correction Processor (BCP)
Volume IV	Data File Control Processor (DFCP)
Volume V	Orbit Determination Executive (ODE)
Volume VI	Convergence Processor (CP)
Volume VII	Differential Correction Module (DCM)
Volume VIII	Data Editing Processor (DEP)
Volume IX	Covariance Matrix Processor (CMP)
Volume X	State Transition Matrix Module (STMM)
Volume XI	Observation Computation Module (OCM)
Volume XII	Measurement Partial Derivative Module (MPDM)
Volume XIII	Residual Computation Processor (RCP)
Volume XIV	Display Processor

VOLUME VII
DIFFERENTIAL CORRECTION MODULE (DCM)

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1.0 CORRELATION TO LEVEL B

This document presents the level C software requirements that satisfy the level B software requirements specified for the differential correction module in the following sections of JSC IN 77-FM-57 (ref. 1), sections 5.7, 6.0 (fig. 6-6), 7.2.6, and 8.3.6.

2.0 GENERAL DESCRIPTION

The differential correction module (DCM) shall be called by the convergence processor (CP) in order to compute corrections to the position, velocity and other specified solve-for parameters by utilizing a weighted least squares fit to the tracking data residuals as follows:

$$\delta \bar{X} = (A^T W A + \Lambda_p^{-1})^{-1} (A^T W \bar{r} + \Lambda_p^{-1} \delta \bar{X}_p)$$

where

- $\delta \bar{X}$ = current change to the estimate of the solution vector
- A = matrix of partial derivatives of the observation with respect to the solution vector parameters
- W = diagonal weighting matrix; each diagonal element is the inverse of the variance associated with respective observation ($W = 1/\sigma^2$)
- Λ_p = a priori covariance matrix
- \bar{r} = residual vector (actual observation minus computed observation)
- $\delta \bar{X}_p$ = vector of differences between the a priori (\bar{X}_p) and current (\bar{X}) estimate of the solution vector ($\delta \bar{X}_p = \bar{X}_p - \bar{X}$)

The term $A^T W A + \Lambda_p^{-1}$ in the above equation is referred to as the normal matrix N , i.e.:

$$N = A^T W A + \Lambda_p^{-1}$$

It can be expressed in terms of the observations as follows:

$$N = \sum_{i=1}^n (\bar{a}_i W_i \bar{a}_i^T) + \Lambda_p^{-1}$$

where

n = the number of observations

$\bar{a}_i = \left[\frac{\partial G_i}{\partial \bar{X}} \right]^T$ = the partials of the i^{th} observation with respect to the solution vector. Note: Throughout this document G is used as a generalized variable to represent the particular observation under consideration.

W_i = the weight of the i^{th} observation

The recursive scheme described below (sec. 3.6) for inverting the symmetric normal matrix N' by diagonalization will yield the inverse of the normal matrix $N^{-1} = \Lambda$ and the correction $\delta \bar{X}$ to the position, velocity and other solution vector parameters. Further, this scheme provides a meaningful result in the cases where the resultant normal matrix is either singular or not positive definite.

When determining a solution, the corrections are computed with respect to the initial estimate (i.e., specified input values) of the solution parameters.

The sequence of mathematical operations required to satisfy this module is presented in the flow chart DCM of appendix A.

The following is a general description of the DCM as shown in the flow chart DCM.

The normal matrix is initialized with the inverse of the a priori covariance matrix.

The current estimate of the solution vector is then input to the FFP to generate a vehicle ephemeris over the observation interval. Ephemeris for all TDRS vehicles in the signal path are provided by the orbit determination executive via the convergence processor.

The following data shall be assembled by the DCM for each batch of tracking data to be processed:

- a. Batch header
- b. Station characteristics
- c. Signal path configuration
- d. Data types contained in batch
- e. Observation weights
- f. Solve-for biases

The DCM shall determine from the batch header the number of valid data frames contained in the batch. These data frames shall be processed sequentially. Within each data frame, all of the observation data types shall be processed one at a time.

The observational data and corresponding ephemeris segments for each data frame shall be obtained from storage. Elevation tests shall be performed on all data frames. Additionally, altitude tests shall be performed for relay data frames. These tests shall be performed only once per data frame. If the computed elevation and/or altitude for a given frame are less than a user specified minimum value, processing of that frame shall be terminated and the DCM shall proceed to the next data frame to continue processing.

For each data frame that passes the elevation/altitude tests the following items shall be computed for each valid observation within the frame:

- a. An estimated observation value and residual
- b. The partial derivative of the computed observation with respect to position, velocity, and bias
- c. A state transition matrix (once per data frame)
- d. The associated row of the partials matrix
- e. The normal matrix of the observation
- f. The updated normal matrix

After the above computations are made for each valid observation within a frame, the DCM shall proceed to the next data frame and continue processing.

After all valid observations within the observation interval have been processed, the normal matrix is inverted and the changes to the solution vector are computed.

Upon completion of the differential correction process, control is returned to the convergence processor.

3.0 FORMULATION OF EACH FUNCTION

3.1 INITIALIZATION

The initialization function shall initialize the normal matrix N with the inverse of the a priori covariance matrix Λ_p^{-1} .

Subsequently, as each observation is processed, the normal matrix shall be accumulated as follows:

$$N = N + \sum_{i=1}^n \bar{a}_i W_i \bar{a}_i^T$$

where

n = the number of observations

\bar{a}_i = the partials of the i^{th} observation with respect to the solution vector

W_i = the weight of the i^{th} observation

Since the normal matrix is symmetric, only the lower triangle portion need be stored. This will reduce the required number of storage locations for a solution vector of k elements from k^2 to $k(k+1)/2$.

The initialization function shall also determine if there are any dynamic parameters (vent and drag) in the solution vector. If so, these values shall be loaded into an auxiliary table to be passed to FFP.

3.2 EPHEMERIS

The observation interval ΔT is defined to be the time interval from the first observation of the first batch to the last observation of the last batch. An ephemeris spanning this interval is required for each vehicle in the signal path. The ephemeris shall be in the Aries-mean-of-1950 coordinate system and shall contain a minimum of nine points with the ephemeris step size Δt between points equivalent to one beta step.

The time interval equivalent to one beta step is determined from the equation

$$\Delta t = \frac{R\beta}{\sqrt{\mu}}$$

where

Δt = time interval in hours (hr)

β = beta step in $(E.r.)^{1/2}$, E.r. = Earth radius

μ = Earth gravitational parameter in $(E.r.)^3/hr^2$

R = position vector magnitude

The observation interval ΔT shall be examined to determine if it is possible to obtain nine or more ephemeris points using an ephemeris step size Δt of one beta step. If nine points can be obtained, the integration interval $\Delta T'$ shall be set equal to the observation interval ΔT . If nine points cannot be obtained, the integration interval $\Delta T'$ shall be set equal to a time interval equivalent to 10 beta steps. Manuevers from the mission plan table (MPT) which lie outside the observation interval shall be ignored in constructing this ephemeris.

The free flight predictor shall use position P_0 and velocity V_0 of the current estimate of the solution vector supplied by the convergence processor (CP) at anchor time t_0 to generate a free flight target vehicle ephemeris spanning the specified integration interval $\Delta T'$. The variables defined in table I shall be provided the free flight predictor and the variables defined in table II shall be returned.

The orbit determination executive (ref. 3) shall provide the required TDRS ephemerides via the convergence processor (ref. 2).

TABLE I.- FREE FLIGHT PREDICTOR INPUT VARIABLES

Variable	Description
t_o	Anchor time associated with specified solution vector and start time of observation interval
\bar{X}_o	M50 state at anchor time
ΔT	Integration interval
α_D	Drag multiplier from solution vector
$\bar{\alpha}_v$	Vent forces from solution vector
INTEG	Integrator force model options

TABLE II.- FREE FLIGHT PREDICTOR OUTPUT VARIABLES

Variable	Description
VEPH(NV)	Vehicle (NV) ephemeris spanning the integration interval which contains the following data at each point: <p data-bbox="500 485 1357 548">t = time of ephemeris point (no further spaced than one beta step)</p> <p data-bbox="500 579 1406 642">\bar{P} = position components (X_v, Y_v, Z_v) in Aries-mean-of-1950 Cartesian coordinate system</p> <p data-bbox="500 674 1406 737">\bar{V} = velocity components (\dot{X}_v, \dot{Y}_v, \dot{Z}_v) in Aries-mean-of-1950 Cartesian coordinate system</p>

3.3 BATCH PROCESSING

The DCM will process one batch at a time in the order encountered in the vehicle data table. For each batch the following functions shall be performed:

- a. Batch header fetch
- b. Station characteristics fetch
- c. Signal path configuration
- d. Data types
- e. Observation weights
- f. Solve-for-bias initialization
- g. Frame initialization

3.3.1 Batch Header Fetch

With the input of a batch number, a systems routine shall locate and return the batch header data as shown in table III.

TABLE III.- BATCH HEADER VARIABLES

Variable	Description
NV	Vehicle identification code
NR	Receiver station identification code
NX	Transmitter station identification code
NT1	Forward link relay satellite identification code
NT2	Return link relay satellite identification code
DT	Data type
V	Reference frequency <ul style="list-style-type: none"> a. S-band direct <ul style="list-style-type: none"> (1) Two-way transmitter frequency from incoming data message (2) Three-way receiving station's estimate of the transmitter frequency from the incoming data message (3) Transmitter direct frequency - user specified value of transmitter frequency to be used for processing three-way direct Doppler data b. Relay - target vehicle (user spacecraft) transmitter frequency
bFp	Relay return link translation frequency
K	Doppler model coefficient <ul style="list-style-type: none"> (1) S-band direct - K is the composite of the target vehicle transponder multiplier and the Doppler extractor multiplier (nominal value is $1000 \times 240/221$). (2) Relay - K is the Doppler extractor multiplier (nominal values are 1000 for S-band and 100 for ku-band).
ω_3	Doppler bias frequency - effective bias frequency used in receiving ground station Doppler counter subsystem
AR	Range ambiguity interval
BT	Batch time - time tag of the first valid observation
NF	Total number of data frames

3.3.2 Station Characteristics Fetch

Utilizing the receiver (NR) and transmitter (NX) station identification codes obtained from the batch header, a systems routine shall locate and return the characteristics of each station as shown in table IV.

TABLE IV.- REQUIRED STATION CHARACTERISTICS VARIABLES

Variable	Description
λ	Geodetic longitude of the station
ϕ_D	Geodetic latitude of the station
r_s	Geocentric radius to the station
z_G	Distance of station from Earth's equatorial plane, positive north
r_G	Radius of the station from Earth's spin axis
$n_o(I)$	Modulus of refraction at the station from the table of monthly refraction information. The month desired is chosen based on the batch time of the batch.
$h_s(I)$	Atmospheric scale height from the table of monthly refraction information
w_R	Range data weight
w_A	Angle data weight
w_{D2}	Two-way Doppler data weight
w_{D3}	Three-way Doppler data weight

3.3.3 Signal Path Configuration

The signal path configuration shall be determined by checking the forward link relay satellite identification code (NT1).

If NT1 = 0, then IDOP = 0, $E_m = E_d$

If NT1 \neq 0, then IDOP = 1, $E_m = E_r$

where

IDOP = signal path configuration flag. The following code is used in this document: 0 = direct, 1 = relay. Direct can be interpreted as C-band or direct S-band tracking types, since IDOP is used to define the signal path configuration rather than the measurement set.

E_m = minimum configuration elevation angle

E_d = user specified minimum direct elevation angle

E_r = user specified minimum relay elevation angle

The sequence of operations needed to satisfy this function is presented in the flow chart SET UP in appendix A.

3.3.4 Refraction

The modulus of refraction (N_o) and scale height (H_s) of each station shall be determined for the month associated with the batch time. The value of these parameters shall not be changed if the number of the month changes within the batch.

3.3.5 Data Types

The data type function shall determine the types (angle 1, angle 2, range, Doppler) of data to be processed. Utilizing the data type (DT) specified in the batch header, the four data type parameters (NDT) shall be set as follows:

If DT = a, then NDT (1) = 1 (azimuth)

NDT (2) = 2 (elevation)

NDT (3) = 7 (range)

NDT (4) = 0 (no Doppler)

If DT = b, then NDT (1) = 3 (north/south X-angle)
 NDT (2) = 4 (north/south Y-angle)
 NDT (3) = 7 (direct range)
 NDT (4) = 8 (2-way direct Doppler)

If DT = c, then NDT (1) = 5 (east/west X-angle)
 NDT (2) = 6 (east/west Y-angle)
 NDT (3) = 7 (direct range)
 NDT (4) = 8 (2-way direct Doppler)

If DT = d, then NDT (1) = 3 (north/south X-angle)
 NDT (2) = 4 (north/south Y-angle)
 NDT (3) = 0 (no range)
 NDT (4) = 8 (three-way direct Doppler)

If DT = e, then NDT (1) = 5 (east/west X-angle)
 NDT (2) = 6 (east/west Y-angle)
 NDT (3) = 0 (no range)
 NDT (4) = 8 (three-way direct Doppler)

If DT = f, then NDT (1) = 0 (no angle 1)
 NDT (2) = 0 (no angle 2)
 NDT (3) = 7 (two-way/three-way relay range)
 NDT (4) = 8 (two-way/three-way relay Doppler)

If DT = g, then NDT (1) = 0 (no angle 1)
 NDT (2) = 0 (no angle 2)
 NDT (3) = 7 (hybrid relay range)
 NDT (4) = 8 (hybrid relay Doppler)

If DT = none of the above, then set error flag IODERR = 32. In the ODE, this value will generate the message:

*** ODE ERROR ***

ERROR IN BATCH DATA TYPE. VEHICLE (XXXX). MODE 1

The sequence of mathematical operations required to satisfy this function is presented in flow chart DTYPE in appendix A.

3.3.6 Observation Weight

The observation weight function shall determine the weight to be used with the four data types. Utilizing the data weights obtained from the station characteristics table (table IV), the observation weight function shall set up the observation weighting array as follows:

$$W(1) = W_A$$

$$W(2) = W_A$$

$$W(3) = W_R$$

$$W(4) = W_{D2} \text{ if data type} = b, c \text{ or } f$$

$$W(4) = W_{D3} \text{ if data type} = d, e \text{ or } g$$

where $W(I)$ = observation weighting array

$I = 1$ (angle 1), $I = 2$ (angle 2), $I = 3$ (range), $I = 4$ (Doppler)

W_A = angle data weight

W_R = range data weight

W_{D2} = two-way/three-way Doppler data weight

W_{D3} = hybrid Doppler data weight

In addition, by setting one or more of the data weight equal to zero, the weighting array shall be used to exclude the corresponding measurement types from the differential correction.

This capability shall be available only in the SB mode. The user shall have the capability to include or exclude from SB processing any of the following measurement types:

- a. C-band azimuth angle
- b. C-band elevation angle
- c. C-band range

- d. S-band direct X-angle
- e. S-band direct Y-angle
- f. S-band direct range
- g. S-band direct two-way Doppler
- h. S-band direct three-way Doppler
- i. Two-way/three-way relay range through TDRSS
- j. Two-way/three-way relay Doppler through TDRSS
- k. Hybrid relay range through TDRSS
- l. Hybrid relay Doppler through TDRSS

Nominally, all data types shall be included. The SB data type include/exclude flags shall be maintained as part of the SBCT. The observation weight function shall use these flags to determine those data types in an assembled data set that, though contained in batches flagged for inclusion by the DFCP, must be excluded from the SB solution in process.

There shall be 12 measurement-type exclusion flags available for specification by the user corresponding to the 12 types of exclusions above. The value 1 will indicate the ON condition while 0 will indicate the OFF condition; i.e., 0 = include, 1 = exclude.

An illustration of the sequence of mathematical operations required to satisfy this function is presented in flow chart WEIGHT in appendix A.

3.3.7 Solve-For Bias Initialization

If the solution vector contains a data bias for the data (two-way/three-way or hybrid relay Doppler) in the batch, set the solve-for bias parameters (b_p) to the current estimate of the value.

The sequence of mathematical operations required to satisfy this function is presented in flow chart SOLFOR in appendix A.

3.3.8 Frame Initialization

Prior to processing the frames of data contained in a batch of data, the following variables shall be initialized as indicated.

$$FC = 1, t' = -10^{50}, ST = -10^{50}, SNR = 0$$

where

- FC = frame counter which will be incremented until the number of frames (NF) specified in the batch header has been processed
- t' = saved frame time which is used to prevent recomputation of variables that are a function of time instead of data type
- ST = saved observation time which is used to prevent recomputation of variables in the observation computation module until the observation time changes
- SNR = saved receiver number which is used to prevent recomputation of variables in the observation computation module until the receiver station number changes

3.4 FRAME PROCESSING

For each frame, the observation data and ephemeris segments shall be obtained. As the data are processed, elevation and RF signal path altitude tests shall be performed. If the computed elevation and altitude values are less than a user specified minimum value, processing of that frame shall be terminated and the DCM shall move on to the next frame to continue processing.

3.4.1 Observation Data

Each frame of observation data shall contain the observation data shown in table V.

TABLE V.- OBSERVATION DATA INPUT VARIABLES

Variable	Description
t_R	Time
D(1)	Angle 1 (A, X)
D(2)	Angle 2 (E, Y)
D(3)	Range
D(4)	Doppler
τ	Doppler sampling interval. This is determined by the rate at which Doppler measurements are read out of the Doppler counter and by the data compression interval.
IE (1)	Edit flag for angle 1
IE (2)	Edit flag for angle 2
IE (3)	Edit flag for range
IE (4)	Edit flag for Doppler

3.4.2 Ephemeris Segments

The ephemeris function shall locate and return nine ephemeris points consisting of time, position (3 elements), and velocity (3 elements) and spanning the observation time (t_R) for each vehicle specified (NV, NT1, NT2) in the batch header in the following manner:

- a. If t_R is prior to the fourth point, then the first nine points of the ephemeris shall be returned.
- b. If t_R is after the fourth point and prior to the last four points of the ephemeris, then the nine points returned shall be centered about t_R .
- c. If t_R is such that there are not four points of the ephemeris with time greater than t_R , then the last nine points of the ephemeris shall be returned.

For direct data (IDOP = 0), the nine points of the target vehicle ephemeris shall be stored in ephemeris array EPH(1).

For relayed data (IDOP = 1), the nine points of the return link relay vehicle TDRS 2 ephemeris shall be stored in ephemeris array EPH(1), the nine points of the target vehicle ephemeris in ephemeris array EPH(2), and the nine points of the forward link relay vehicle TDRS 1 ephemeris in ephemeris array EPH(3).

The sequence of mathematical operations required to satisfy this function is presented in flow chart EPHEMS in appendix A.

3.4.3 Refraction Model Validity Tests

Following the first call to the OCM for the current data frame the DCM shall perform elevation angle and signal path altitude tests. The purpose of these tests is to exclude from DC processing those observations that occur in the regions where the atmospheric model inaccuracy degrades the observation modeling beyond reasonable limits. In each of these tests, if the computed value is less than the user-specified minimum value, processing of the data frame shall be terminated and the next frame shall be fetched.

The elevation angle tests are:

- a. In the case of two-way direct data and two-way/three-way relayed data, the OCM-computed elevation of the vehicle with respect to the receiving station horizon shall be compared with the user specified minimum elevation value.
- b. In the case of three-way direct data and hybrid relayed data, the OCM computed elevation of the vehicle with respect to both the receiving and transmitting station horizons shall be compared with the user-specified minimum elevation value.

The RF signal path altitude test:

For relayed data, if the target vehicle is known to be on the Earth farside (with respect to the pertinent TDRS) portion of the orbital are via the forward relay test $R(3) \cdot R(4) < |R(3)|^2$ or return relay test $R(3) \cdot R(2) < |R(3)|^2$, the respective relay satellite to target vehicle RF signal path altitude of closest approach shall be computed once per data frame as follows:

$$h_F = \sqrt{\frac{|\bar{R}(4)|^2 |\bar{R}(3)|^2 - [\bar{R}(4) \cdot \bar{R}(3)]^2}{|\bar{R}(4)|^2 + |\bar{R}(3)|^2 - 2 [\bar{R}(4) \cdot \bar{R}(3)]}} \quad -r_e$$

$$h_R = \sqrt{\frac{|\bar{R}(2)|^2 |\bar{R}(3)|^2 - [\bar{R}(2) \cdot \bar{R}(3)]^2}{|\bar{R}(2)|^2 + |\bar{R}(3)|^2 - 2 [\bar{R}(2) \cdot \bar{R}(3)]}} \quad -r_e$$

where

h_F = altitude of forward link relay

h_R = altitude of return link relay

$\bar{R}(2)$ = return link relay satellite position in the M50 Cartesian coordinate system from the OCM

$\bar{R}(3)$ = vehicle position in the M50 Cartesian coordinate system from the OCM

$\bar{R}(4)$ = forward link relay satellite position in the M50 Cartesian coordinate system from the OCM

r_e = equatorial radius of the reference ellipsoid (system parameter)

If the relayed data passes through a significant amount of the Earth atmosphere, the data shall not be used to update the solution vector. Therefore, if either the computed forward or return link relay altitude is less than the user specified minimum value, terminate further processing of the data frame. Figure 1 illustrates the relayed RF signal path geometry. The sequence of mathematical operations required for RF signal path altitude computation is presented in flow chart RALT in appendix A.

3.4.4 Observation Initialization

Prior to processing the observation data contained in a frame, initialize the data type counter I so that Doppler data will be processed first; i.e., $I = 4$.

Subsequently, as each data type is processed, the type counter shall be decremented until all four types of data specified by the data type parameters (NDT) have been processed.

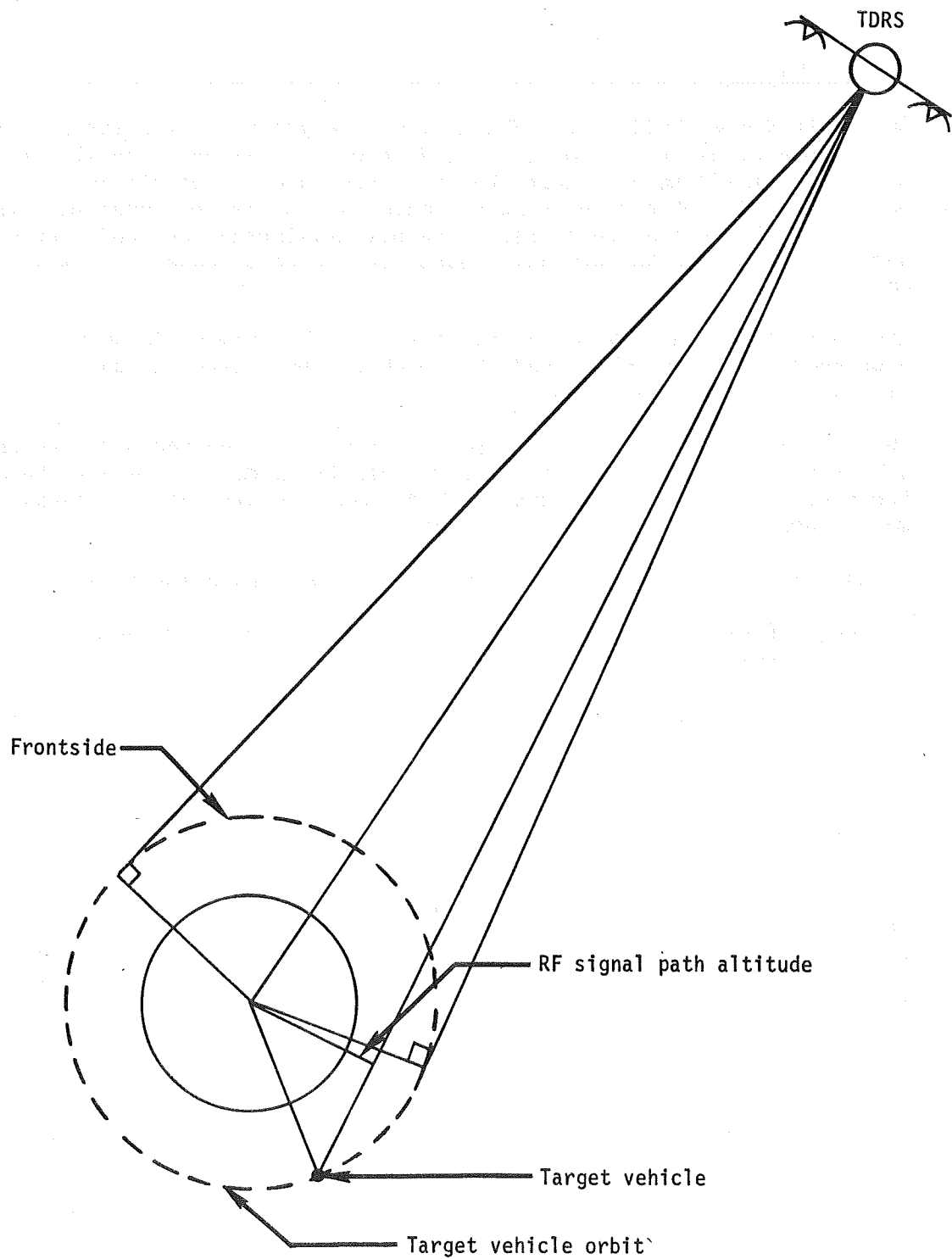


Figure 1.- Relay RF signal path geometry.

3.5 OBSERVATION PROCESSING

3.5.1 General

Each data frame shall have a fixed record length and a consistent ordering of data types within the record. Fill data will be inserted by the low speed input processor (LSIP) where meaningful data types are not available. For example, relay data will at most consist of range and Doppler measurements since the TDRS ground station angle measurements are not meaningful for orbit determination use. In this case the two angle words in the relay data frame will contain fill data.

One observation data type will be processed at a time. As each data type is processed, the data value shall be checked. Data with an edit flag or filler data will not be processed.

On each differential correction pass, a test shall be performed to ensure that all four types of observation data defined for a data frame have been considered for processing. After the entire frame has been processed, determine if any more frames of data are to be processed.

As each valid observation is processed, the DCM calls the following modules:

- a. OCM (ref. 4) to compute an estimated observation value and residual for the observaton
- b. MPDM (ref. 5) to compute the partial derivative of the observation with respect to position, velocity and solve-for biases (if included in the solution vector)
- c. STMM (ref. 6) to compute the partial derivatives of position and velocity with respect to initial position and velocity, and any dynamic parameters. The STMM is called only once per data frame

The DCM accumulates the partials matrix and the normal matrix as each data type is processed. As each data frame is read in, the frame time t_r is saved and used to avoid recomputation of parameters that need to be computed only once per data frame.

3.5.2 Setup To Process Observation

To process an observation, variables utilized by the OCM (ref. 4) shall be defined as follows:

$$G_o = D(I)$$

$$IDT = NDT(I)$$

where G_o = observation value

$D(I)$ = data value obtained from frame

$D(1)$ = angle 1

$D(2)$ = angle 2

$D(3)$ = range

$D(4)$ = Doppler

IDT = data type indicator

IDT = 1 (azimuth)

2 (elevation)

3 (north/south X-angle)

4 (north/south Y-angle)

5 (east/west X-angle)

6 (east/west Y-angle)

7 (range)

8 (Doppler)

NDT(I) = data type parameter

NDT(1) denotes angle 1

NDT(2) denotes angle 2

NDT(3) denotes range

NDT(4) denotes Doppler

3.5.3 Observation Computation

The observation computation module (ref. 4) shall be used to obtain (1) an estimated observation value, (2) the observation residual, (3) the elevation of the vehicle with respect to the receiving and transmitting stations, (4) the time and position of each point in the signal path at the time the signal is transmitted from that vehicle or satellite for both the start and end times of the measurement interval, (5) the velocity of the Orbiter vehicle at the start and end time of the Doppler measurement interval, and (6) the position of the vehicle with respect to the station in the topodetic coordinate system. The

variables defined in table VI shall be provided to the OCM and the variables defined in table VII shall be returned.

TABLE VI.- DCM to OCM INTERFACE

DCM Parameter Vol. VII, table VI	OCM Parameter ^a	OCM Vol. XI, Section	Unit	Description
IDOP	IDOP	3.1,3.2,3.3	Flag	1 = relay 0 = direct
EPH(I)	EPH(I)	3.4	Internal	9-point ephemeris tables
X _λ X _{rG} X _{ZG} X _{φD}	λ _X r _{GX} Z _{GX} φ _{DX}	3.4.5	Internal	Transmitter location parameter
r _λ r _{rG} r _{ZG} r _{φD}	λ _R r _{GR} Z _{GR} φ _{DR}	3.4.5	Internal	Receiver location parameter
X _{N_D} X _{H_S}	N _X H _X	3.5	Internal	Transmitter refr. modulus and scale height
r _{N_D} r _{H_S}	N _R H _R	3.5	Internal	Receiver refr. modulus and scale height
t _R	t _R	3.1,3.2,3.3	Internal	Measurement time at receiver
G _O	GRNG GDOP GANG1 GANG2	3.1,3.2,3.3	Internal Hz Internal Internal	Measurement value
A _R	A _R	3.1	Internal	Range ambiguity interval
τ	τ	3.2	Internal	Doppler count interval
ν	ν _{NX}	3.2	H _Z	Reference frequency
K	K	3.2	Internal	Frequency multiplier
ω ₃	ω ₃	3.2	H _Z	Offset frequency in Doppler

^aSee table VI, vol. XI.

TABLE VI.- Concluded

DCM Parameter Vol.VII, table VI	OCM Parameter	OCM Vol. XI, Section	Unit	Description
BD	b_D	3.2	H_z	Relay Doppler bias (solve for)
BF_P	(bF_P)	3.2	H_z	Return link TDRS translation frequency
IDT	IDT	Appendix	Flag	Measurement type I D
NR	NR	Appendix	Flag	Current receiver I D
NX	NX	Appendix	Flag	Current transmitter I D
SNR	SNR	Appendix	Flag	Receiver ID from previous call
ST	ST	Appendix	Internal	Measurement time from previous call

TABLE VII.- $\boxed{\text{OCM}}$ to $\boxed{\text{DCM}}$ INTERFACE

OCM Parameter	DCM Parameter Vol. VII appendix	OCM Vol. XI, Section	Unit	Description
$\vec{R}_p^{(e)}, t_p^{(e)}$	$R(p), t(p)$		Internal	M50 position and epoch of participation of each point on signal path (corresponding to measurement time, t_R)
$\vec{V}_Q^{(e)}$	\bar{V}	3.1,3.2, 3.3	Internal	Vehicle velocity (M50) at vehicle participation
$\vec{R}_p^{(s)}, t_p^{(s)}$	$\bar{R}'(p), t'(p)$		Internal	M50 position and epoch of participation of each point on signal path (corresponding to start time of Doppler count interval, at receiver $t_1^{(s)} = t_R - \tau$)
$\vec{V}_Q^{(s)}$	\bar{V}'	3.2	Internal	Vehicle velocity (M50) at vehicle participation
$\Delta t(p)^{(e)}$	$\Delta t(p)$	3.1,3.2, 3.3	Internal	Signal delay times for each (p^{th}) signal path leg (corresponding to measurement time, t_Q)
$\Delta t(p)^{(s)}$	$\Delta t'(p)$	3.2	Internal	Signal delay times for each (p^{th}) signal path leg (corresponding to start time of Doppler count interval)
$E(1)$	$E(I)$	3.5	Internal	Elevation angle for receiver leg
S	$G(3)$	2.1	Internal	Computed range measurement
f	$G(4)$	3.2	Hz	Computed Doppler measurement
A, E	$G(1), G(2)$	3.3	Internal	Computed angle measurement

TABLE VII.- Concluded

OCM Parameter	DCM Parameter Vol. VII appendix	OCM Vol. XI, Section	Unit	Description
$X_{n/s}, Y_{n/s}$	G(1), G(2)	3.3	Internal	Computed angle measurement
$X_{E/W}, Y_{E/W}$	G(1), G(2)	3.3	Internal	Computed angle measurement
D(s)	r	3.1	Internal	Measurement residuals (observed-computed)
D(f)		3.2		
D(A), D(E)		3.3		
$D(X_{n/s}),$ $D(Y_{n/s})$				
$D(X_{E/W}),$ $D(Y_{E/W})$				
$\vec{R}_{T/D} =$ (ξ, η, ζ)	$R_{TD}(1)$	3.3	Internal	Receiver leg range vector in ENU topodetic coordinate (for angle measurements)
ρ^2	ρ^2	3.3	Internal	Square of magnitude of receiver leg range vector (for angle measurements)
A	A	3.3	Internal	Transformation matrix: from TEI to ENU (for angle measurements)
RNP	RNP	3.3	Internal	Transformation matrix: from M50 to TEI (for angle measurements)

3.5.4 Measurement Partial Derivative Computation

The measurement partial derivative module (ref. 5) shall be used to obtain the partial derivatives of the observation with respect to the (1) vehicle position, (2) vehicle velocity, and (3) observation biases. The partial derivatives of the observations with respect to all other dynamic parameters included in the solution vector is zero. The variables defined in table VIII shall be provided to the MPDM and the variables defined in table IX shall be returned.

TABLE VIII.- ! DCM ! to ! MPDM ! INTERFACE

DCM Parameter	MPDM Parameter	MPDM Vol. XI, Section	Unit	Description
A	A	3.1.4	-	TEI to ENU transformation matrix
$R_{TD}(1)$	$\vec{R}_{TD}(1) = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$	3.1	E.r.	Relative position of vehicle with respect to the receiver in ENU coordinates
\vec{R}_p, t_p	\vec{R}_p, t_p		E.r., hr	M50 cartesian coordinates of each point on the RF signal path
\vec{R}_p, t_p	$\vec{R}_p^{(e)}, t_p^{(e)}$		E.r., hr	The (e) denotes at end the Doppler counting interval.
\vec{R}_p', t_p'	$\vec{R}_p^{(s)}, t_p^{(s)}$	3.1.5, 3.1.6	E.r., hr	(s) denotes at the start of the Doppler count interval
Δt_p	Δt_p	3.1.5	hr	Signal delay time on the p^{th} leg of the RF signal path.
Δt_p	$\Delta t_p^{(e)}$	3.1.6	hr	(e) denotes at the end of the Doppler count interval
$\Delta t_p'$	$\Delta t_p^{(s)}$	3.1.6	hr	(s) denotes at the start of the Doppler count interval
ρ^2	ρ^2	3.1	E.r.	$\xi^2 + \eta^2 + \zeta^2$ = square of the geometric distance from the receiver to the first participant (TDRS or vehicle)
IDOP	IDOP	3.1	Flag	Signal path configuration
IDT	IDT	4.0	Flag	Data type indicator
K	K	3.1.6	--	Doppler model multiplier

TABLE VIII.- Concluded

DCM Parameter	MPDM Parameter	MPDM Vol. XI, Section	Unit	Description
ν	ν_{NX}	3.1.6	Hz	Reference frequency
τ	τ	3.1.6	hr	Doppler count interval
t'	t	4.0	hr	Observation time

TABLE IX.- $\boxed{\text{MPDM}}$ to $\boxed{\text{DCM}}$ INTERFACE

MDDM Parameter	DCM Parameter	MPDM Vol. XI, Section	Unit	Description
$\frac{\partial G}{\partial R_v}$	$\frac{\partial G}{\partial R_v}$	3.1	Internal	Partial of observation with respect to M50 position
$\frac{\partial G}{\partial V_v}$	$\frac{\partial G}{\partial V_v}$	3.2	Internal	Partial of observation with respect to M50 velocity
$\frac{\partial G}{\partial b}$	$\frac{\partial G}{\partial b}$	3.3	Internal	Partial of observation with respect to bias

3.5.5 State Transition Matrix Computation

The state transition matrix submodule (ref. 6) shall be used to obtain the partial derivatives of position and velocity at the vehicle time (t_v) with respect to (1) the anchor position and velocity of the vehicle and (2) the dynamic solution vector parameters once per data frame. The vehicle time (t_v) and position (\bar{R}_v) is a function of the signal path configuration. The value of these parameters is determined as follows:

For direct tracking data (IDOP = 0): $t_v = t(2)$, $\bar{R}_v = \bar{R}(2)$

For relayed tracking data (IDOP = 1): $t_v = t(3)$, $\bar{R}_v = \bar{R}(3)$

where. $t(p)$ = time of point p on the signal path as determined by the light time algorithm of the OCM

$\bar{R}(p)$ = position of point p on signal path as determined by the light time algorithm of the OMC

The DCM shall request a state transition matrix $\phi(t, t_0)$ whenever t is:

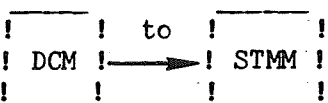
- a. The vehicle time associated with the current frame time
- b. An ephemeris time
- c. A vent on time
- d. A vent off time

To effect this computation, the DCM shall provide:

- a. M50 state and time at which the state transition matrix is required.
- b. M50 state and time of previous STM request.
- c. State transition matrix from the previous request.
- d. State vector solve-for flags that identify which dynamic parameters are present
- e. Drag multiplier from current state vector
- f. Vent flags that specify on and off times for each vent

In addition to saving the most recently computed state transition matrix, the DCM shall save the state transition matrix, the vehicle state vector, and time tag associated with the STMM call immediately preceding the batch time of the next batch, in case the current batch overlaps the next batch. This is for purposes of reinitializing the state transition matrix module of the first frame of the next batch.

The variables defined in table X shall be provided to the STMM and the variables defined in table XI shall be returned.

TABLE X.-  INTERFACE

DCM Parameter Vol.VII, table I	STMM Parameter	Unit	Description
Link ID	Link ID	Flag	Identifies DC link
$\vec{R}_L, \vec{V}_L, t_L$	X_L, t_L	Internal	M50 state and epoch at initialization time
R, V, t	X, t	Internal	M50 state and epoch at output time
$\phi(t_L, t_0)$	$\phi(t_L, t_0)$	Internal	State transition matrix that maps from anchor time(t_0) to initialization time (t_L)
SVFLGS	Sol. vector content	Flag	Identifies solve-for dynamic parameters and biases in the solution vector.
VNT(J)	VNT_J^a	Flag	Specifies whether J^{th} vent is ON or OFF for current computation

TABLE XI.-  INTERFACE

STMM Parameter Vol. X, Sec. 3.2	DCM Parameter Vol. VII, table I	Unit	Description
$\phi(t, t_0)$	$\phi(t, t_0)$	Internal	State transition matrix, $6 \times (6+M)$, where M = number of dynamic parameters, which maps from anchor time (t_0) to current time (t).

3.5.6 Observation Partial Matrix Computation

The partial derivative of each observation with respect to the solution vector parameters shall be computed using the partial derivative of the observation with respect to the solution vector and the state transition matrix. The partials matrix is formed as follows:

$$\bar{\partial} = \left[\begin{pmatrix} \frac{\partial G}{\partial X} & \frac{\partial X}{\partial X_J} \end{pmatrix} , \bar{b} \right]^T$$

where $\bar{\partial}$ = a column vector of partial derivatives of the observation with respect to the parameters in the solution vector

$\frac{\partial G}{\partial X}$ = a row vector of partial derivatives of the observation with respect to the current vehicle position and velocity (see fig. 2)

$\frac{\partial X}{\partial X_J}$ = a rectangular state transition matrix of partial derivatives of the current vehicle position and velocity with respect to the dynamic parameters in the solution vector at anchor time (t_0) (see fig. 2)

\bar{b} = a row vector of partial derivatives of the observation with respect to the measurement biases in the solution vector

$b_j = 1$, if solution bias j applies to this measurement

= 0, otherwise. If measurement biases are not included in the solution vector, then \bar{b} is omitted from the partials matrix.

$$\begin{aligned}
 \bar{a} &= \left[\frac{\partial G}{\partial \bar{x}} \quad \frac{\partial X}{\partial \bar{x}_j} \right]^T, \quad \bar{b} \\
 \bar{a} &= \left[\begin{array}{c} \frac{\partial G}{\partial \bar{x}_v} \quad \frac{\partial G}{\partial Y_v} \quad \frac{\partial G}{\partial Z_v} \quad \frac{\partial G}{\partial X_v} \quad \frac{\partial G}{\partial Y_v} \quad \frac{\partial G}{\partial Z_v} \\ \frac{\partial \bar{x}_v}{\partial X_o} \quad \frac{\partial \bar{x}_v}{\partial Y_o} \quad \frac{\partial \bar{x}_v}{\partial Z_o} \quad \frac{\partial \bar{x}_v}{\partial X_o} \quad \frac{\partial \bar{x}_v}{\partial Y_o} \quad \frac{\partial \bar{x}_v}{\partial Z_o} \\ \frac{\partial Y_v}{\partial X_o} \quad \frac{\partial Y_v}{\partial Y_o} \quad \frac{\partial Y_v}{\partial Z_o} \quad \frac{\partial Y_v}{\partial X_o} \quad \frac{\partial Y_v}{\partial Y_o} \quad \frac{\partial Y_v}{\partial Z_o} \\ \frac{\partial Z_v}{\partial X_o} \quad \frac{\partial Z_v}{\partial Y_o} \quad \frac{\partial Z_v}{\partial Z_o} \quad \frac{\partial Z_v}{\partial X_o} \quad \frac{\partial Z_v}{\partial Y_o} \quad \frac{\partial Z_v}{\partial Z_o} \\ \frac{\partial \dot{x}_v}{\partial X_o} \quad \frac{\partial \dot{x}_v}{\partial Y_o} \quad \frac{\partial \dot{x}_v}{\partial Z_o} \quad \frac{\partial \dot{x}_v}{\partial X_o} \quad \frac{\partial \dot{x}_v}{\partial Y_o} \quad \frac{\partial \dot{x}_v}{\partial Z_o} \\ \frac{\partial \dot{y}_v}{\partial X_o} \quad \frac{\partial \dot{y}_v}{\partial Y_o} \quad \frac{\partial \dot{y}_v}{\partial Z_o} \quad \frac{\partial \dot{y}_v}{\partial X_o} \quad \frac{\partial \dot{y}_v}{\partial Y_o} \quad \frac{\partial \dot{y}_v}{\partial Z_o} \\ \frac{\partial \dot{z}_v}{\partial X_o} \quad \frac{\partial \dot{z}_v}{\partial Y_o} \quad \frac{\partial \dot{z}_v}{\partial Z_o} \quad \frac{\partial \dot{z}_v}{\partial X_o} \quad \frac{\partial \dot{z}_v}{\partial Y_o} \quad \frac{\partial \dot{z}_v}{\partial Z_o} \end{array} \right] \\
 &\quad \left[\begin{array}{c} \frac{\partial X_v}{\partial X_o} \quad \frac{\partial X_v}{\partial Y_o} \quad \frac{\partial X_v}{\partial Z_o} \quad \frac{\partial X_v}{\partial X_o} \quad \frac{\partial X_v}{\partial Y_o} \quad \frac{\partial X_v}{\partial Z_o} \\ \frac{\partial Y_v}{\partial X_o} \quad \frac{\partial Y_v}{\partial Y_o} \quad \frac{\partial Y_v}{\partial Z_o} \quad \frac{\partial Y_v}{\partial X_o} \quad \frac{\partial Y_v}{\partial Y_o} \quad \frac{\partial Y_v}{\partial Z_o} \\ \frac{\partial Z_v}{\partial X_o} \quad \frac{\partial Z_v}{\partial Y_o} \quad \frac{\partial Z_v}{\partial Z_o} \quad \frac{\partial Z_v}{\partial X_o} \quad \frac{\partial Z_v}{\partial Y_o} \quad \frac{\partial Z_v}{\partial Z_o} \\ \frac{\partial \dot{x}_v}{\partial X_o} \quad \frac{\partial \dot{x}_v}{\partial Y_o} \quad \frac{\partial \dot{x}_v}{\partial Z_o} \quad \frac{\partial \dot{x}_v}{\partial X_o} \quad \frac{\partial \dot{x}_v}{\partial Y_o} \quad \frac{\partial \dot{x}_v}{\partial Z_o} \\ \frac{\partial \dot{y}_v}{\partial X_o} \quad \frac{\partial \dot{y}_v}{\partial Y_o} \quad \frac{\partial \dot{y}_v}{\partial Z_o} \quad \frac{\partial \dot{y}_v}{\partial X_o} \quad \frac{\partial \dot{y}_v}{\partial Y_o} \quad \frac{\partial \dot{y}_v}{\partial Z_o} \\ \frac{\partial \dot{z}_v}{\partial X_o} \quad \frac{\partial \dot{z}_v}{\partial Y_o} \quad \frac{\partial \dot{z}_v}{\partial Z_o} \quad \frac{\partial \dot{z}_v}{\partial X_o} \quad \frac{\partial \dot{z}_v}{\partial Y_o} \quad \frac{\partial \dot{z}_v}{\partial Z_o} \end{array} \right] \\
 &\quad \left[\begin{array}{c} \frac{\partial X_v}{\partial P_1} \quad \frac{\partial X_v}{\partial P_2} \quad \frac{\partial X_v}{\partial P_m} \\ \frac{\partial Y_v}{\partial P_1} \quad \frac{\partial Y_v}{\partial P_2} \quad \frac{\partial Y_v}{\partial P_m} \\ \frac{\partial Z_v}{\partial P_1} \quad \frac{\partial Z_v}{\partial P_2} \quad \frac{\partial Z_v}{\partial P_m} \\ \frac{\partial \dot{x}_v}{\partial P_1} \quad \frac{\partial \dot{x}_v}{\partial P_2} \quad \frac{\partial \dot{x}_v}{\partial P_m} \\ \frac{\partial \dot{y}_v}{\partial P_1} \quad \frac{\partial \dot{y}_v}{\partial P_2} \quad \frac{\partial \dot{y}_v}{\partial P_m} \\ \frac{\partial \dot{z}_v}{\partial P_1} \quad \frac{\partial \dot{z}_v}{\partial P_2} \quad \frac{\partial \dot{z}_v}{\partial P_m} \end{array} \right] \\
 &\quad \left[\frac{\partial G}{\partial b_1}, \dots, \frac{\partial G}{\partial b_n} \right]
 \end{aligned}$$

where m = number of dynamic parameters in the solution vector

n = number of measurement biases in the solution vector

Figure 2.- Partial matrix computation.

3.5.7 Normal Matrix Computation

As each observation is processed, the normal matrix N shall be updated using the observation partials vector \bar{a} and the observation weight W . Since the resulting normal matrix is symmetric, forming the matrix row by row and storing it in lower triangular form will save storage and computing time. It is not necessary to compute the elements past the diagonal of the matrix. The normal matrix shall be updated as follows:

$$N = N + \bar{a} W(I) \bar{a}^T$$

where

N = the normal matrix

\bar{a} = a column vector of partial derivatives of the observation with respect to the parameters in the solution vector

$W(I)$ = the scalar weight of observation data type I . $I = 1$ (angle 1),
 $I = 2$ (angle 2), $I = 3$ (range), $I = 4$ (Doppler)

The current sum of squares of residuals and the predicted residuals shall be accumulated as follows

$$S = \delta X_p^T \Lambda_p^{-1} \delta X_p + \bar{r}^T \bar{w} \bar{r}$$

$$P = \Lambda_p^{-1} \delta X_p + A^T \bar{w} \bar{r}$$

The sequence of operations necessary to execute this function is presented in the flow chart NORMAT of appendix A.

3.6 SOLUTION REQUIREMENTS

After all observations have been processed, the normal matrix N will contain the following data:

$$N = \sum_{i=1}^n \bar{a}_i w_i \bar{a}_i^T + \Lambda_p^{-1} = A^T W A + \Lambda_p^{-1}$$

The corrections $\delta \bar{X}$ to the state vector are then computed according to the equation:

$$\delta \bar{X} = (A^T W A + \Lambda_p^{-1})^{-1} (A^T W \bar{r} + \Lambda_p^{-1} \delta \bar{X}_p)$$

where

$\delta \bar{X}$ = current change to the estimate of the solution vector

A = matrix of partial derivatives of the observations with respect to the solution vector parameters

W = diagonal weighting matrix; each element is the inverse of the variance associated with the respective observation

Λ_p = a priori covariance matrix

\bar{r} = residual vector (actual observation minus computed observation)

$\delta \bar{X}_p$ = vector of differences between the current \bar{X} and a priori (\bar{X}_p) estimate of the solution vector $\delta \bar{X}_p = \bar{X}_p - \bar{X}$

The following is a recursive inversion technique for symmetric matrices. This technique provides a meaningful test for matrix singularity (or ill conditioning) by setting the appropriate element of the solution vector correction to zero, thereby selecting a set of linearly independent parameters for the solution vector. In this case, set MSG = 03.

The basic equations for this inversion technique are

$$S N S^T = D \text{ and } N^{-1} = S^T D^{-1} S$$

where

N = input symmetric matrix

S = a lower triangular matrix with -1 for each diagonal element

D = a diagonal matrix

Let

$$N_i = \begin{bmatrix} N_{i-1} & B_i \\ B_i^T & C_i \end{bmatrix}, \quad S_i = \begin{bmatrix} S_{i-1} & 0 \\ W_i^T & -1 \end{bmatrix} \quad \text{and} \quad D_i = \begin{bmatrix} D_{i-1} & 0 \\ 0 & \alpha_i \end{bmatrix}$$

where

N_{i-1} = upper left $(i-1) \times (i-1)$ submatrix of N

B_i^T = first $i-1$ elements of the i -th row of N

C_i = i^{th} diagonal element of N

S_{i-1} = upper left $(i-1) \times (i-1)$ submatrix of S

W_i^T = first $i-1$ elements of the i -th row of S

D_{i-1} = upper left $(i-1) \times (i-1)$ submatrix of D

α_i = i^{th} diagonal element of D

It can be shown that

$$W_i = N_{i-1}^{-1} B_i = S_{i-1}^T D_{i-1}^{-1} S_{i-1} B_i$$

and

$$\alpha_i = C_i - B_i^T W_i$$

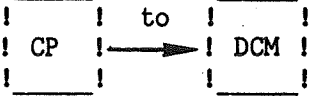
which provide the recursive procedure for calculating S and D with $i \geq 2$ and the initial values $S_1 = -1$ and $D_1 = N_1$.

To obtain a meaningful solution, if N is singular or "ill conditioned," perform the following: If $C_i = 0$ or $\alpha_i/C_i < \epsilon$, set $1/\alpha_i = 0$ for computation of D^{-1} , where ϵ is a small number (system parameter). The net effect of the above is "not compute" a change to the i^{th} element of the solution vector.

The sequence of mathematical operations required for the inversion scheme is presented in flow chart SYMIN of appendix A. The flow chart assumes the input matrix has been collapsed to only the lower triangular portion, which has been stored by rows, e.g., a_{11} , a_{21} , a_{22} , a_{31} , ..., a_{nn} .

The variables shown in table XII shall be provided the inversion scheme.


The variables shown in table XIII shall be returned by the inversion scheme.

TABLE XII.-  INTERFACE

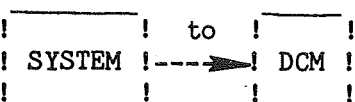
CP Parameter ^a Vol VI, sec. 6.0	DCM Parameter	DCM (Vol. VII), Section	Unit	Description
BC	BC			Batch counter
A_p^{-1}	A_p^{-1}	3.1		Inverse of a priori covariance
X	X	4.0		Current solution vector
Data batches	Data batches	Appendix		Tracking data
SCT	SCT	Appendix		Station characteristics tables
VEPH(I)	VEPH(I)	Appendix		Ephemerides for vehicle and TDRS satellites
BTIMES	BTIMES			Initial times of all batches being processed
h_M	h_M	Appendix		Minimum TDRS signal path altitude
E_D	E_D	3.4.3		Minimum direct elevation angle
E_R	E_R	3.4.3		Minimum relayed elevation angle
DTXCL	DTXCL			Data type exclusion flags
n	n	4.0		Length of solution vector
NB	NB	Appendix		Number of batches being processed
ΔT	ΔT	4.0	hr	Length of data interval

TABLE XII.- Concluded

CP Parameter ^a Vol VI, sec. 6.0	DCM Parameter	OCM (Vol. VII), Section	Unit	Description
Xp	Xp	2.0		A priori state vector
SVFLGS	SVFLGS			Solution vector flags; i.e., flags which indicate the type of dynamic parameters in the solution vector
INTEG	INTEG			Integrator force model options for FFP
TVNT	TVNT			Vent start times
SVNT	SVNT			Vent stop times
LINK ID	LINK ID			Vehicle identifier
δX_p	δX_p			Total differential correction
μ	μ		E.r. ³ / hr ²	Earth gravitational parameter
β	β		(E.r.) ¹ / ₂	Beta step
r_e	r_e		E.r.	Earth radius

TABLE XIII.-  INTERFACE

DCM Parameter	CP Parameter	Unit	Description
X_L	X_L		M50 state at last state transition matrix computation
t_L	t_L		Time of last state transition matrix computation
$Q(t_L, t_0)$	$Q(t_L, t_0)$		State transition matrix at time t_L
Λ	Λ		Updated covariance matrix
IODERR	IODERR		OD error flag
δX	δX		Differential correction
RTWR	RTWR		Weighted sum of squares of residuals
ATWR	ATWR		Predicted residuals
VEPH(NV)	VEPH(NV)		Ephemeris of vehicle
VNT	VNT		Vent flags

TABLE XIV.-  INTERFACE

System Parameter	DCM Parameter	Unit	Description
μ	μ	ER^3/HR^2	Earth gravitational parameter
β	β	$ER^{1/2}$	Beta step
r_e	r_e	ER	Radius of reference ellipsoid

4.0 REFERENCES

1. OPS MCC Ground Navigation Program, Level B, Preliminary Orbit Determination Processing, Formulation Requirements. JSC IN 77-FM-57 (JSC-13647), Oct. 1977.
2. OPS MCC Ground Navigation Program, Level C, Orbit Determination Processing, Formulation Requirements - Volume VI: Convergence Processor (CP). JSC IN 78-FM-30 (JSC-14266), June 1978.
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7. MCC OD Matrices. TRW 77:2511, 4-43, Nov. 4, 1977.
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APPENDIX

FLOW CHARTS FOR
DIFFERENTIAL CORRECTION MODULE

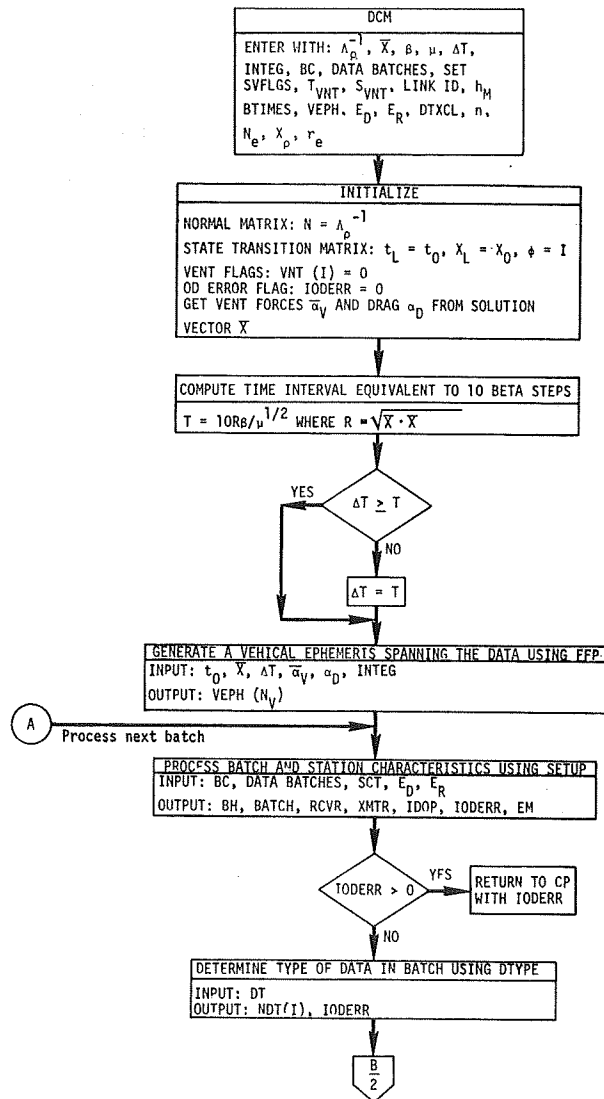


Figure A-1.- Flow diagram for differential correction module.

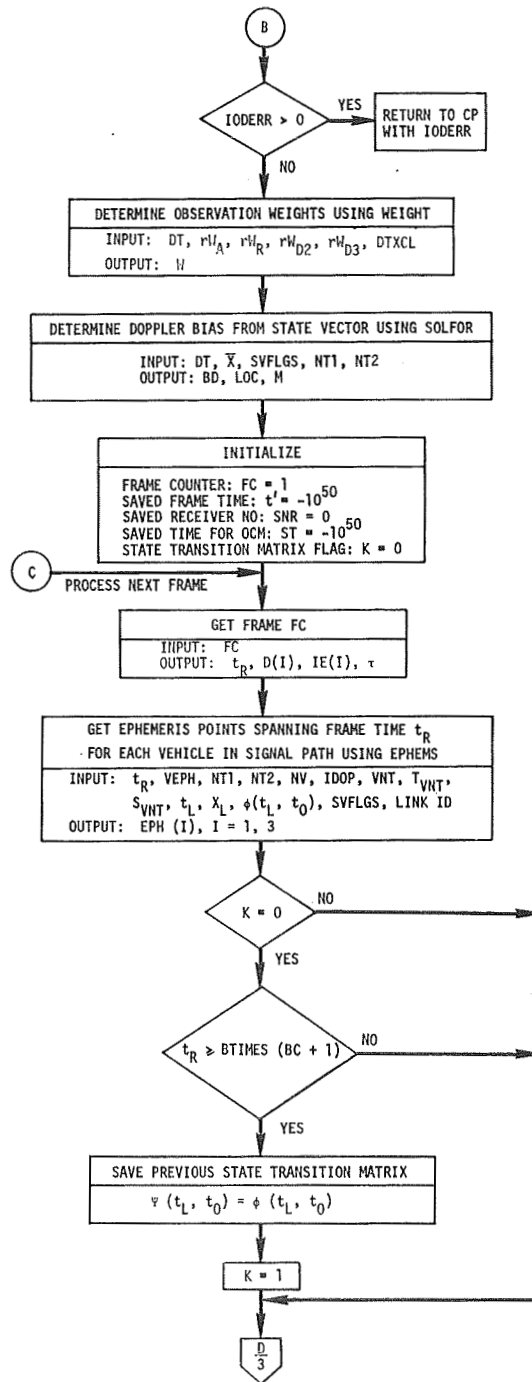


Figure A-1.- Continued

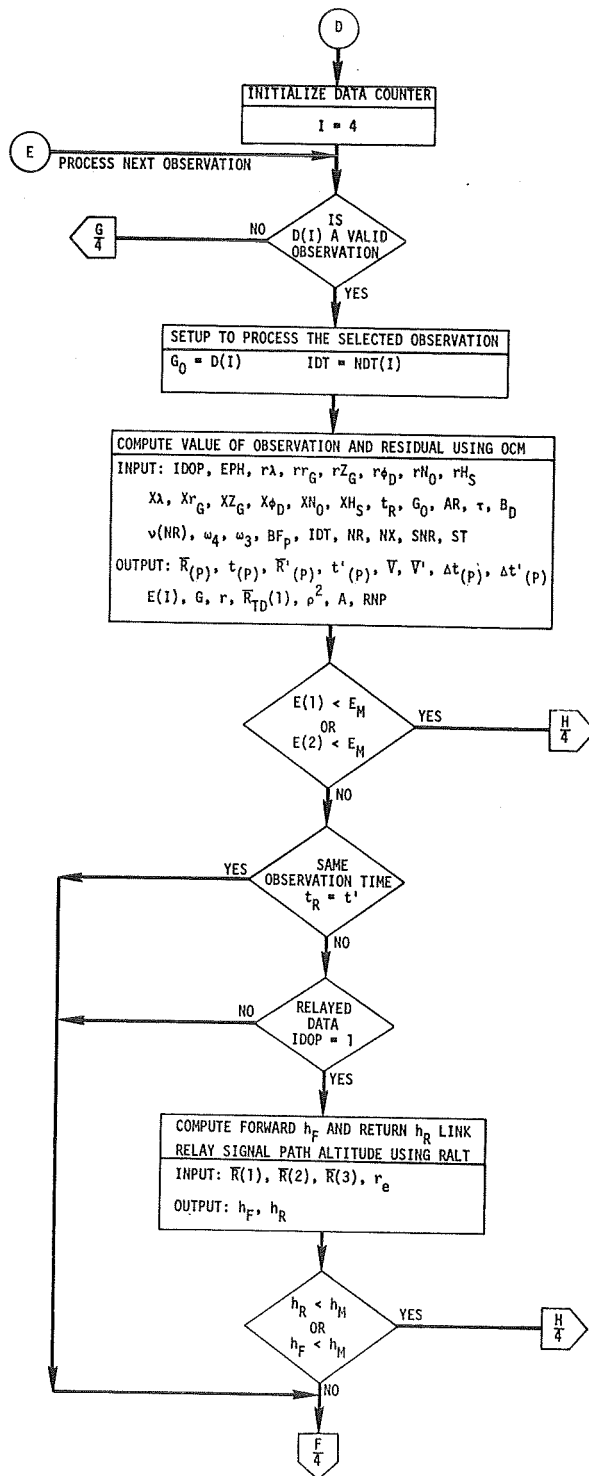
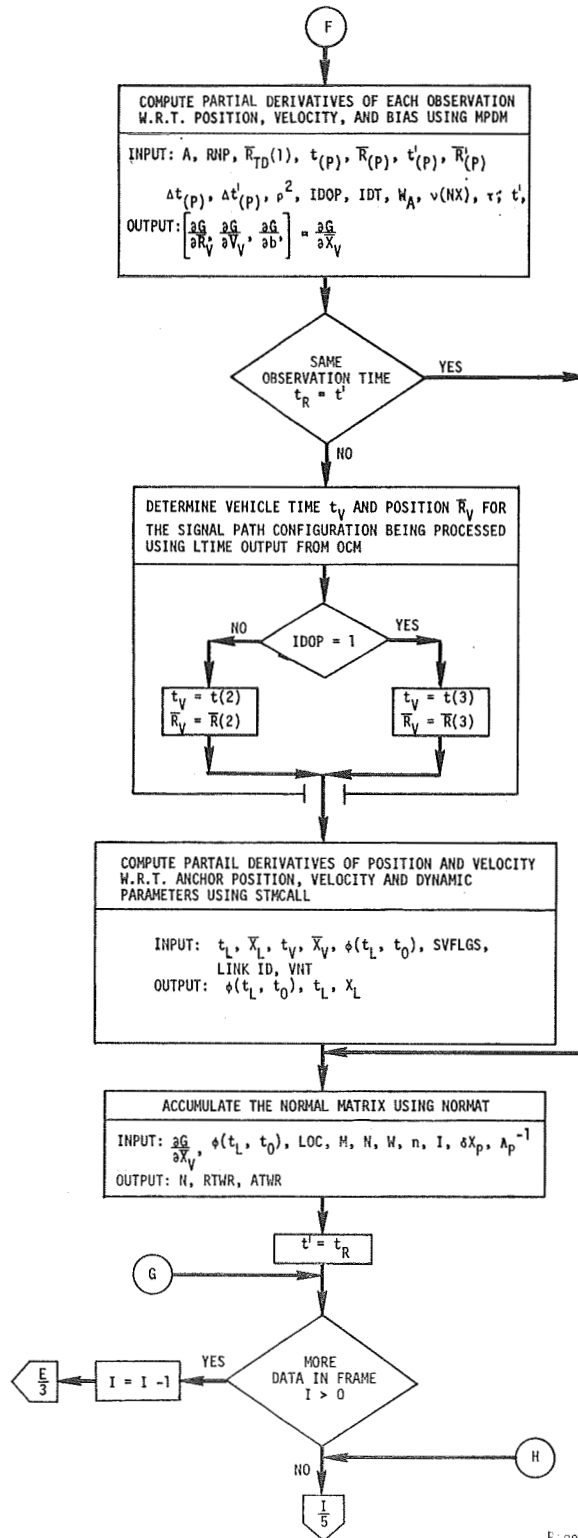


Figure A-1.- Continued.



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Figure A-1.- Continued.

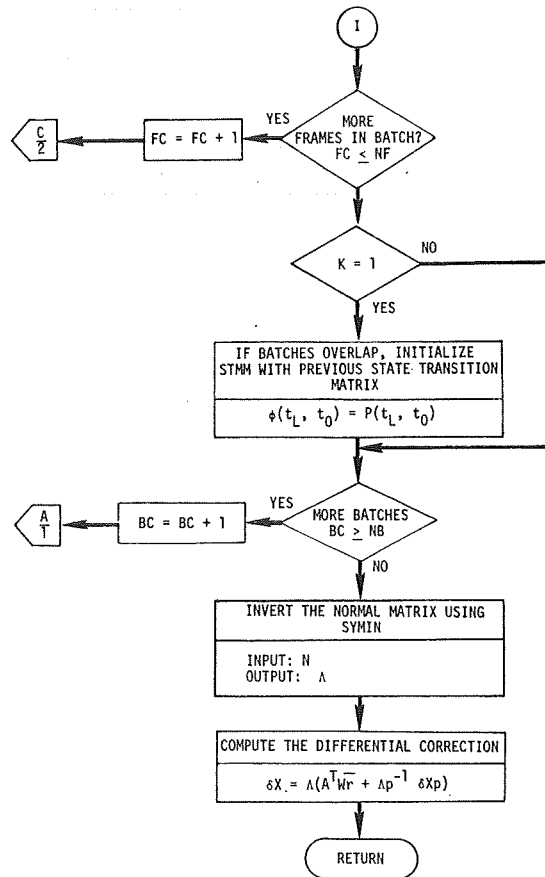


Figure A-1.- Concluded

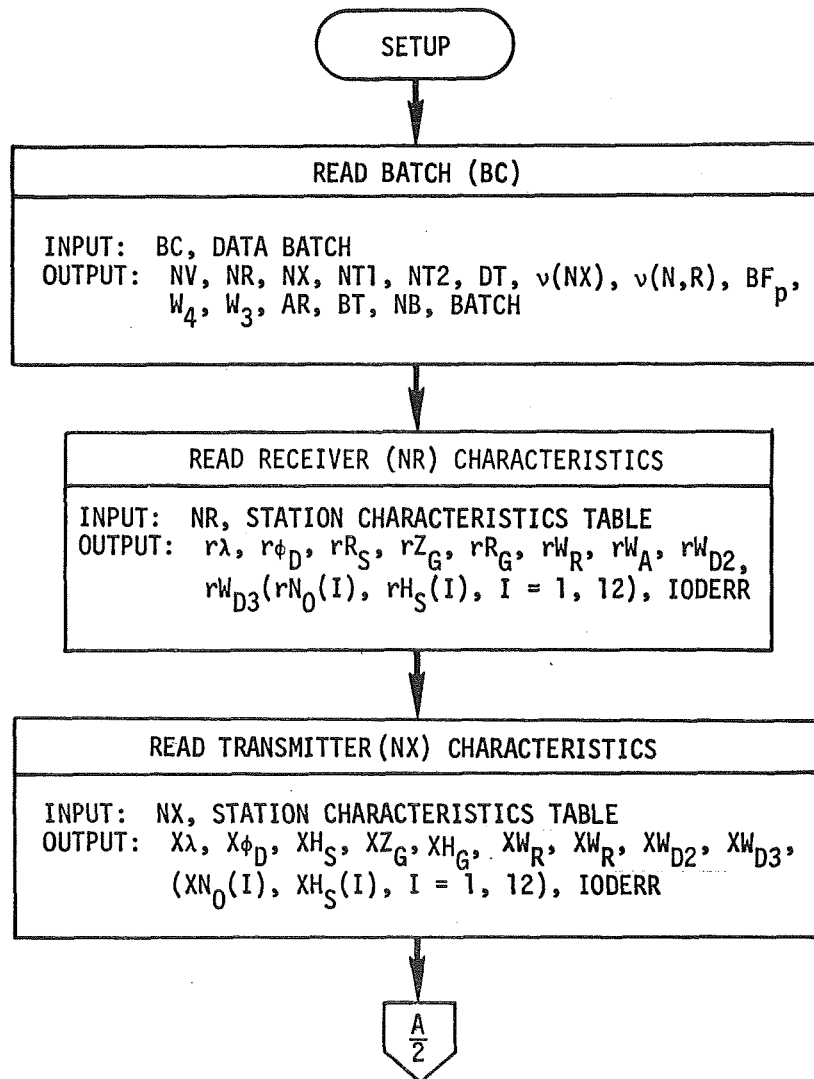


Figure A-2.- Flow diagram for SETUP process

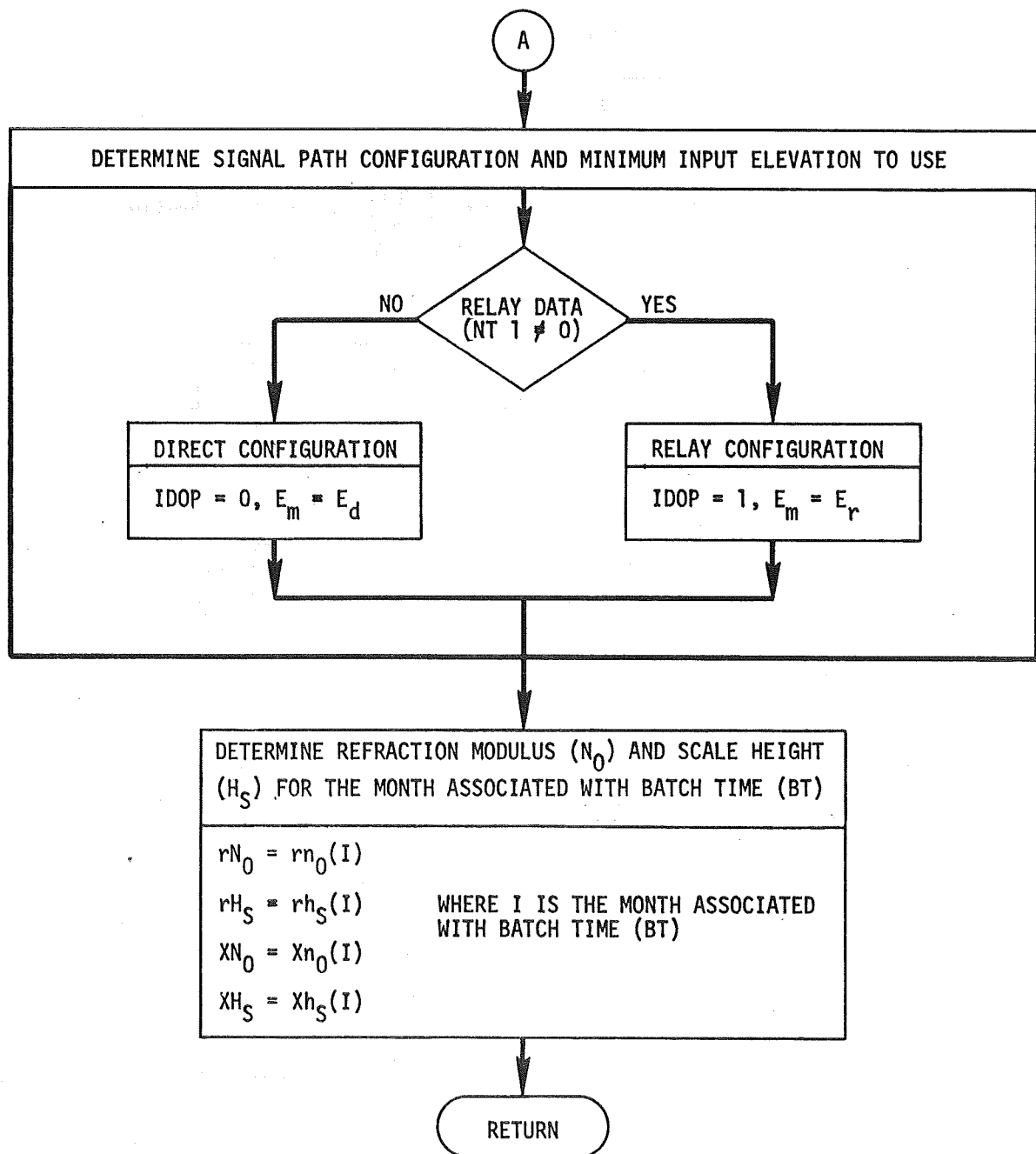


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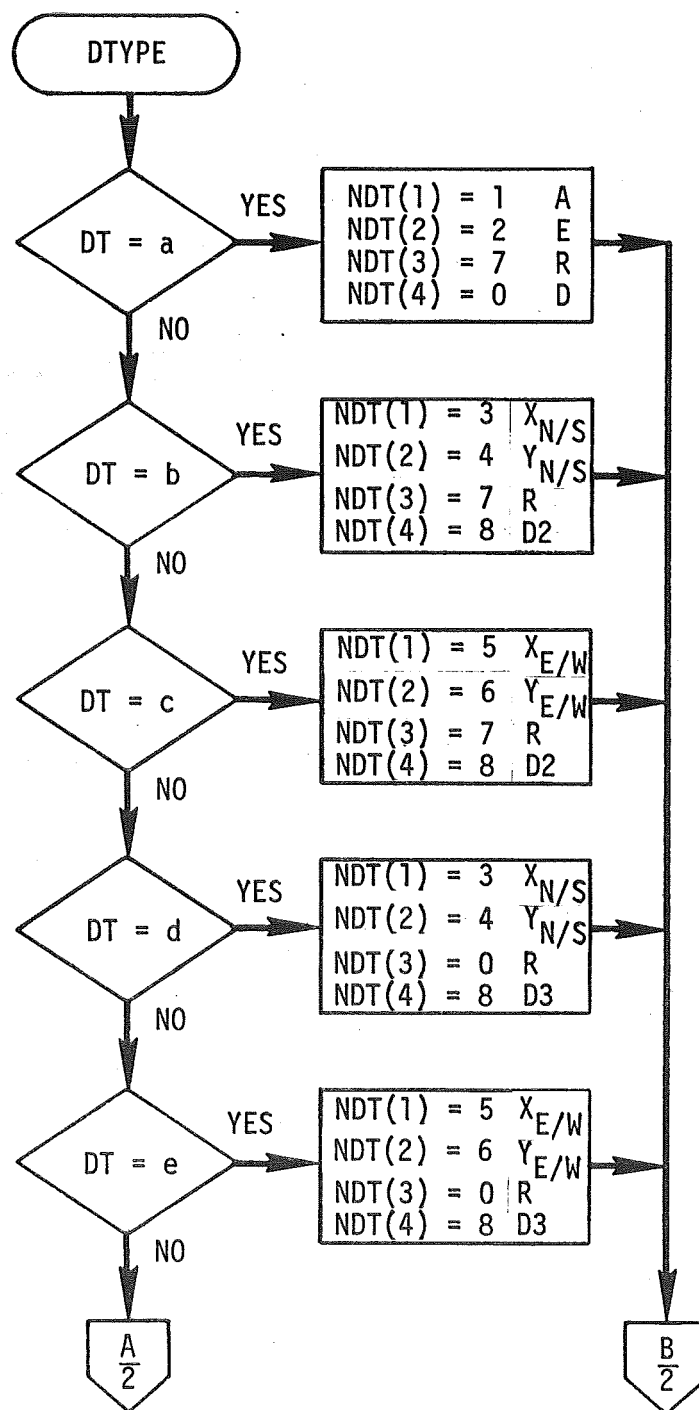


Figure A-3.- Flow diagram: data type (DTYPE) to be processed.

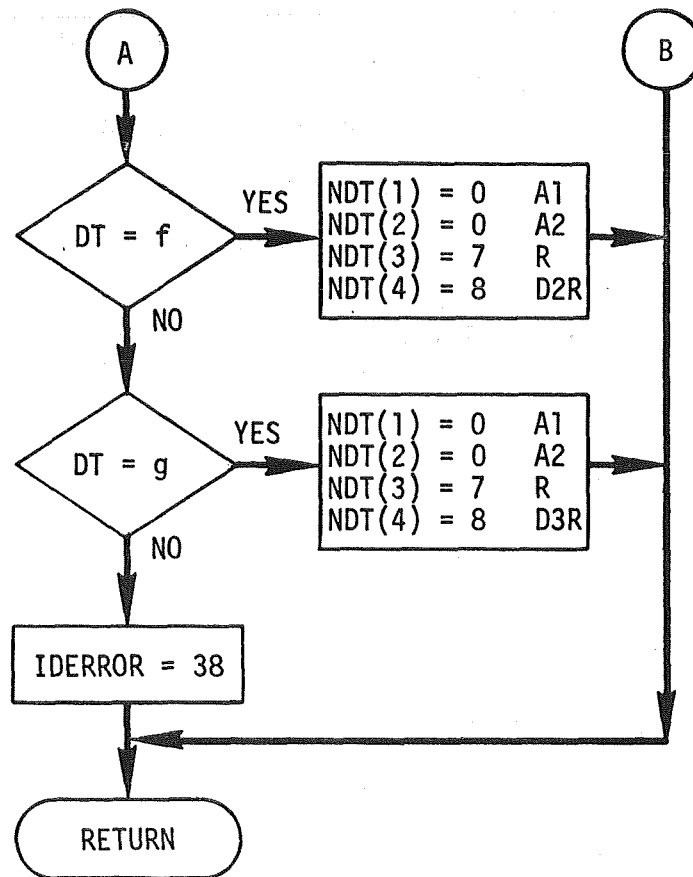


Figure A-3.- Concluded.

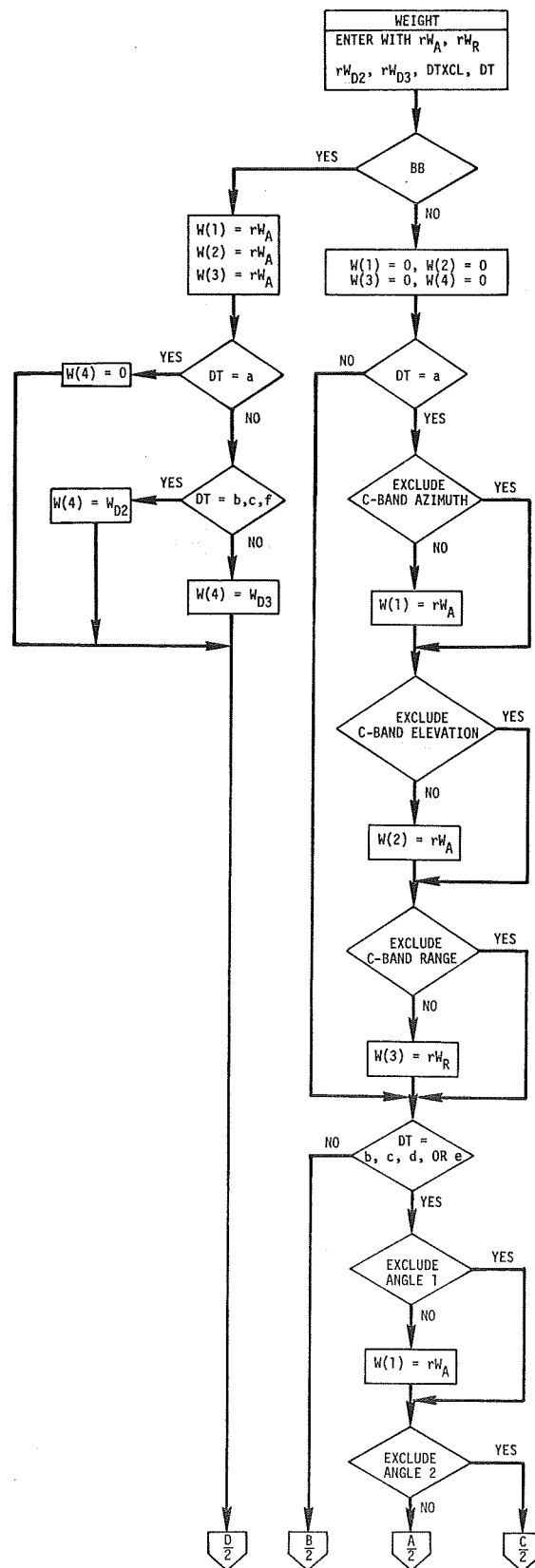


Figure A-4.- Flow diagram for weight process.

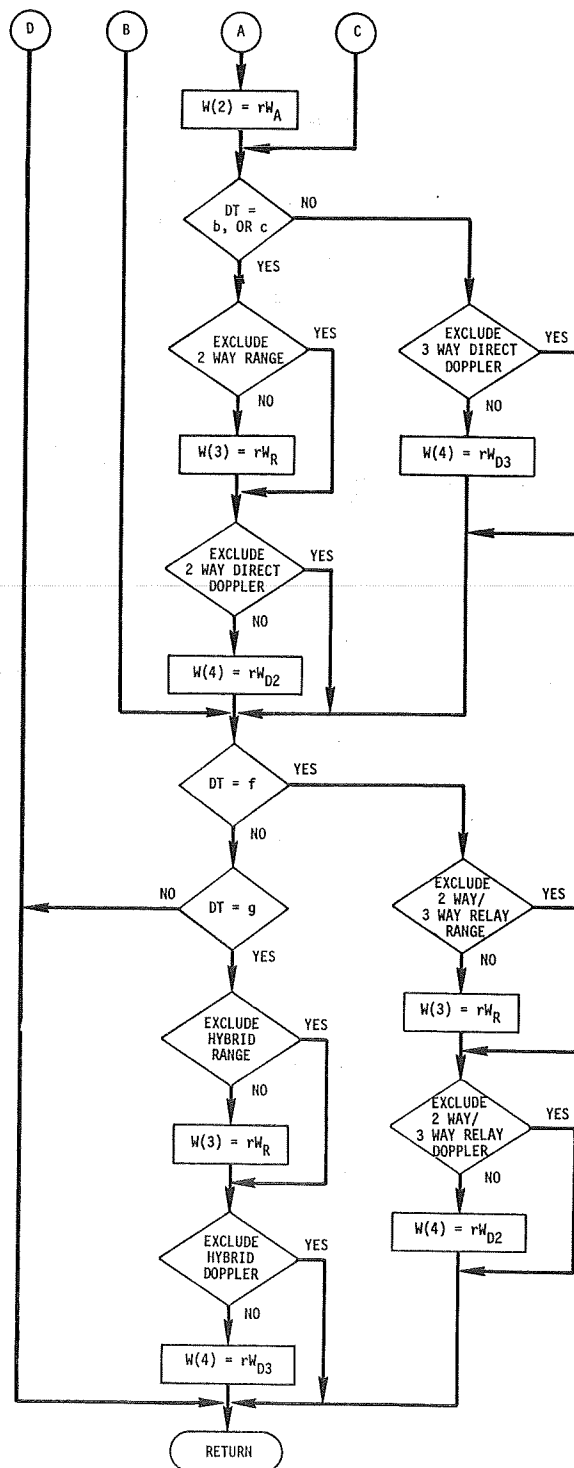
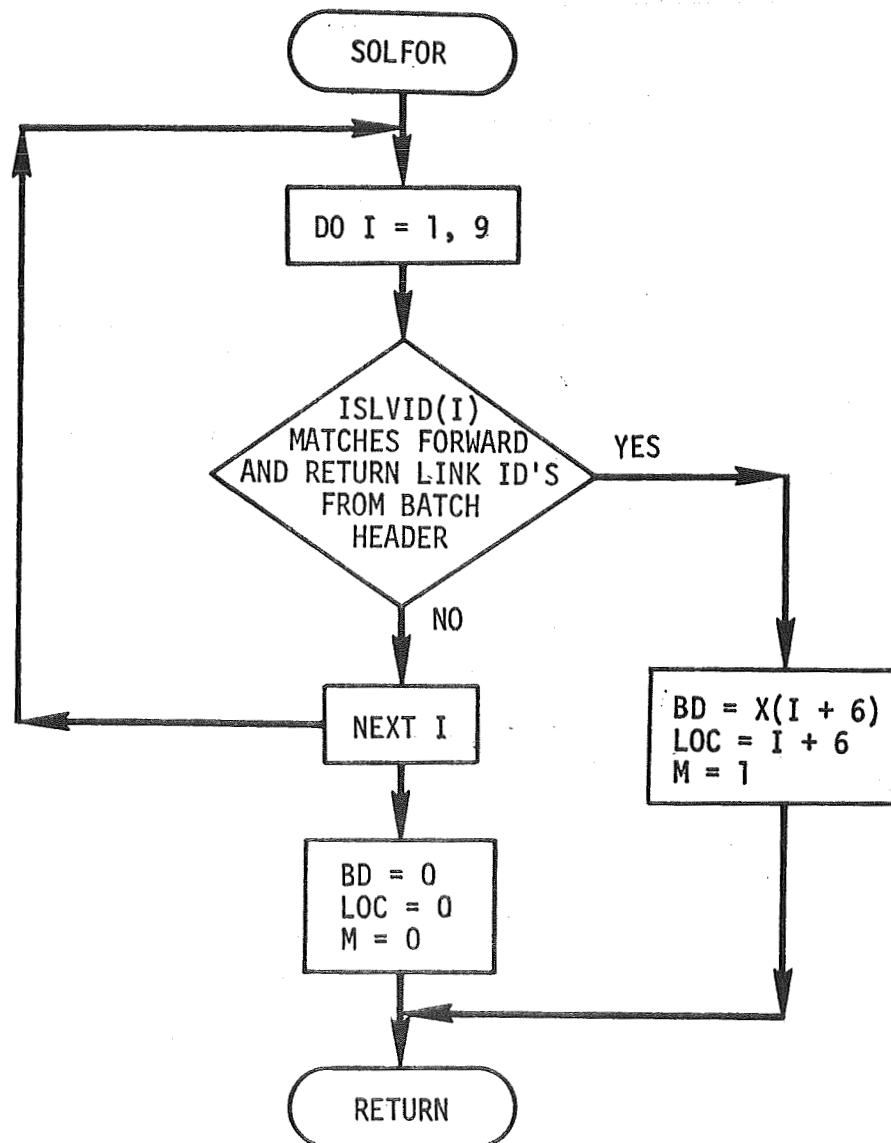


Figure A-4.- Concluded.



\bar{X} - SOLUTION VECTOR
 ISLVID - SOLUTION VECTOR CONTENTS
 BD - DOPPLER BIAS
 LOC - LOCATION IN NORMAL MATRIX TO PUT $\frac{\partial G}{\partial b}$
 M - MULTIPLIER FOR $\frac{\partial G}{\partial b}$

Figure A-5.- Flow diagram for SOLFUR process.

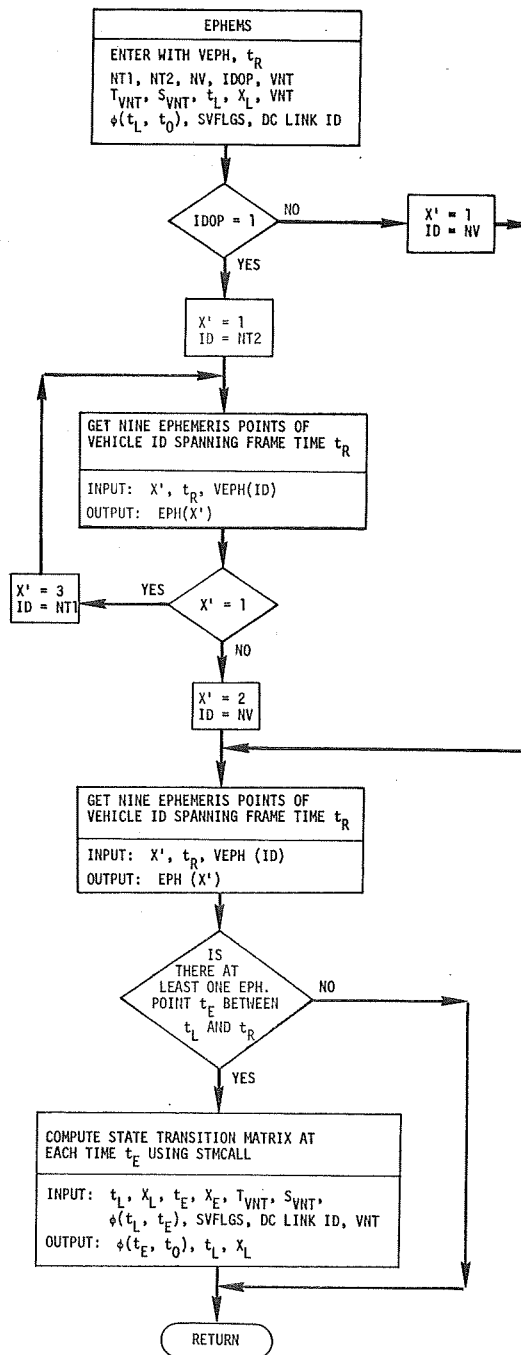
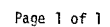


Figure A-6.- Flow diagram for EPHEMS process.



A-16

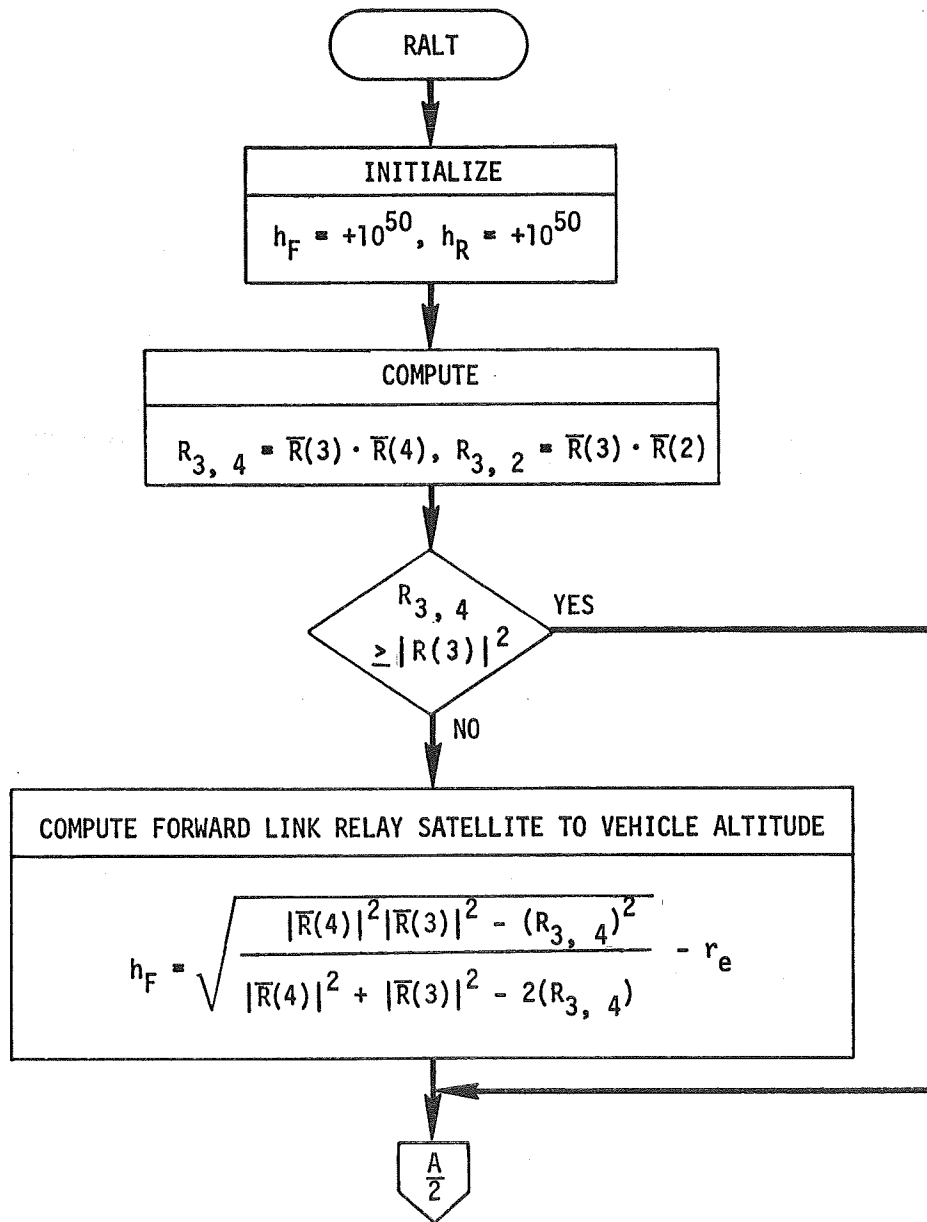


Figure A-8.- Flow diagram: relayed data altitude (RALT) computation.

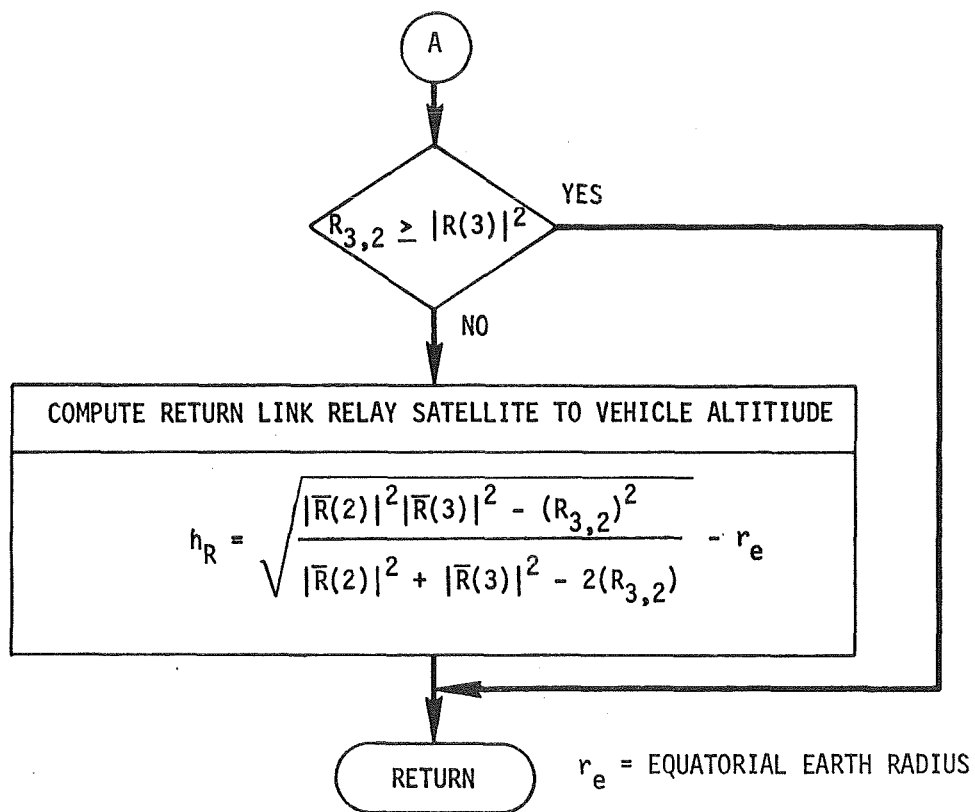


Figure A-8.- Concluded.

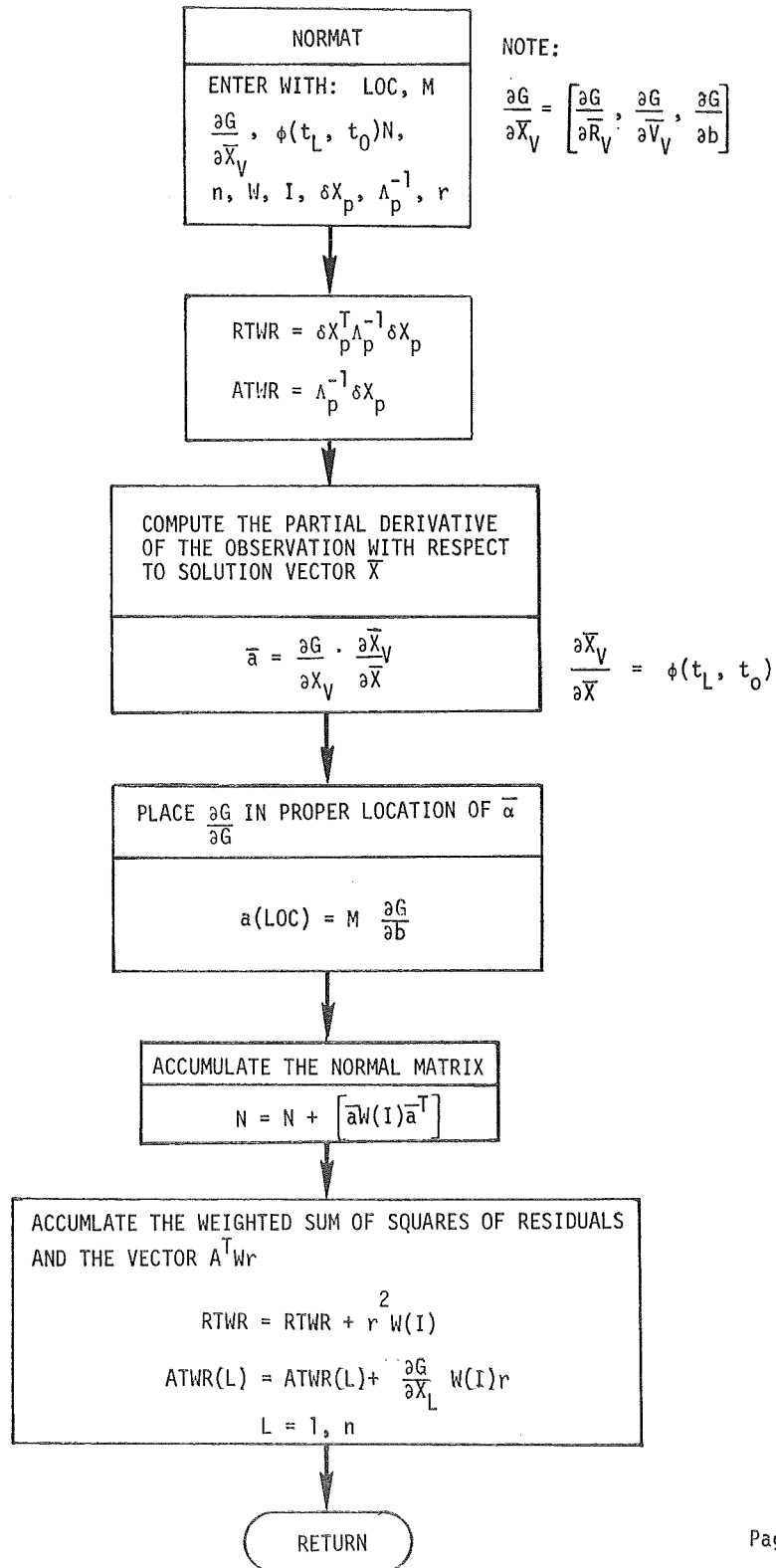


Figure A-18.- Flow diagram for NORMAT process.

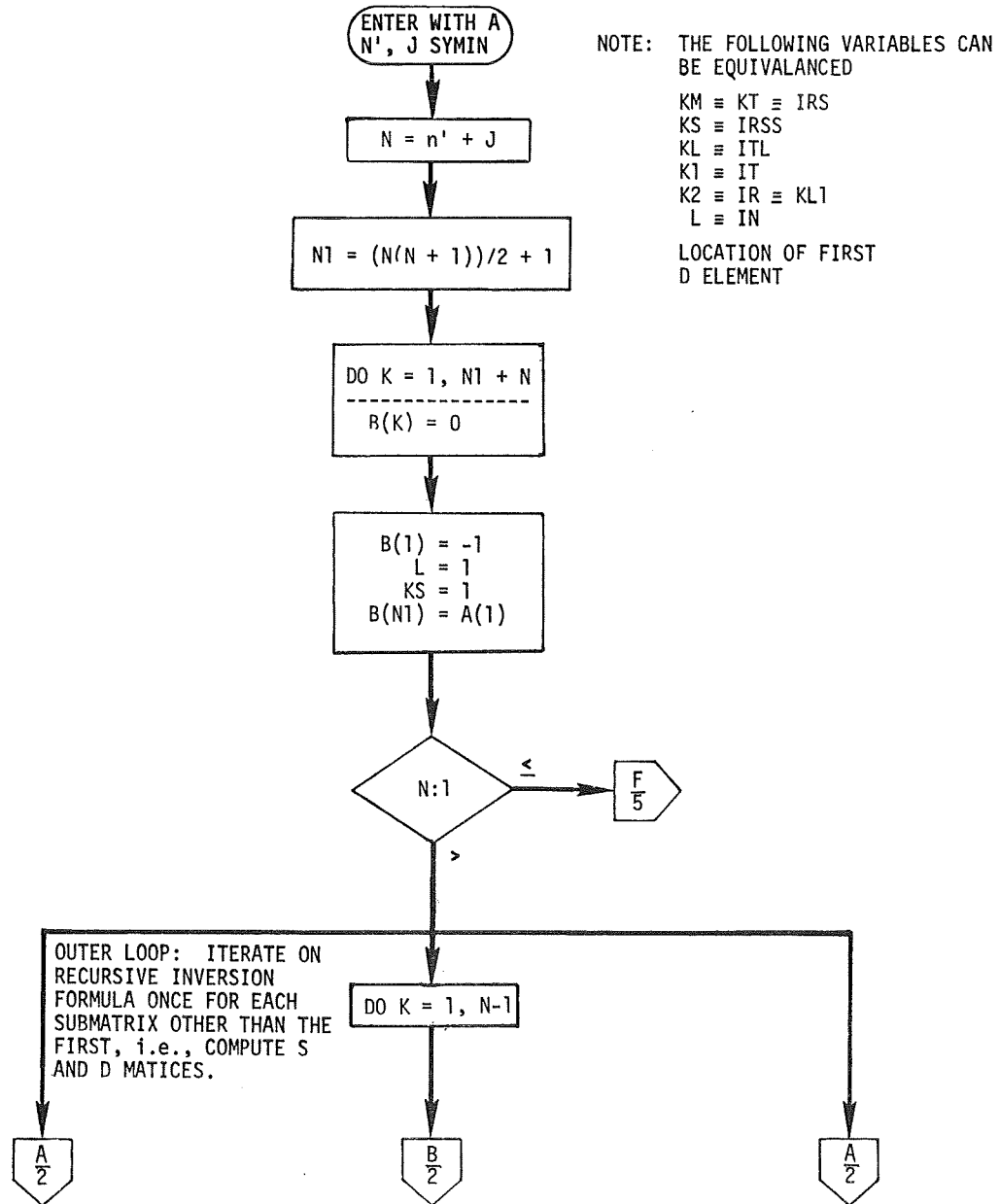


Figure A-19.- Flow diagram; invert normal matrix and compute changes (SYMIN).

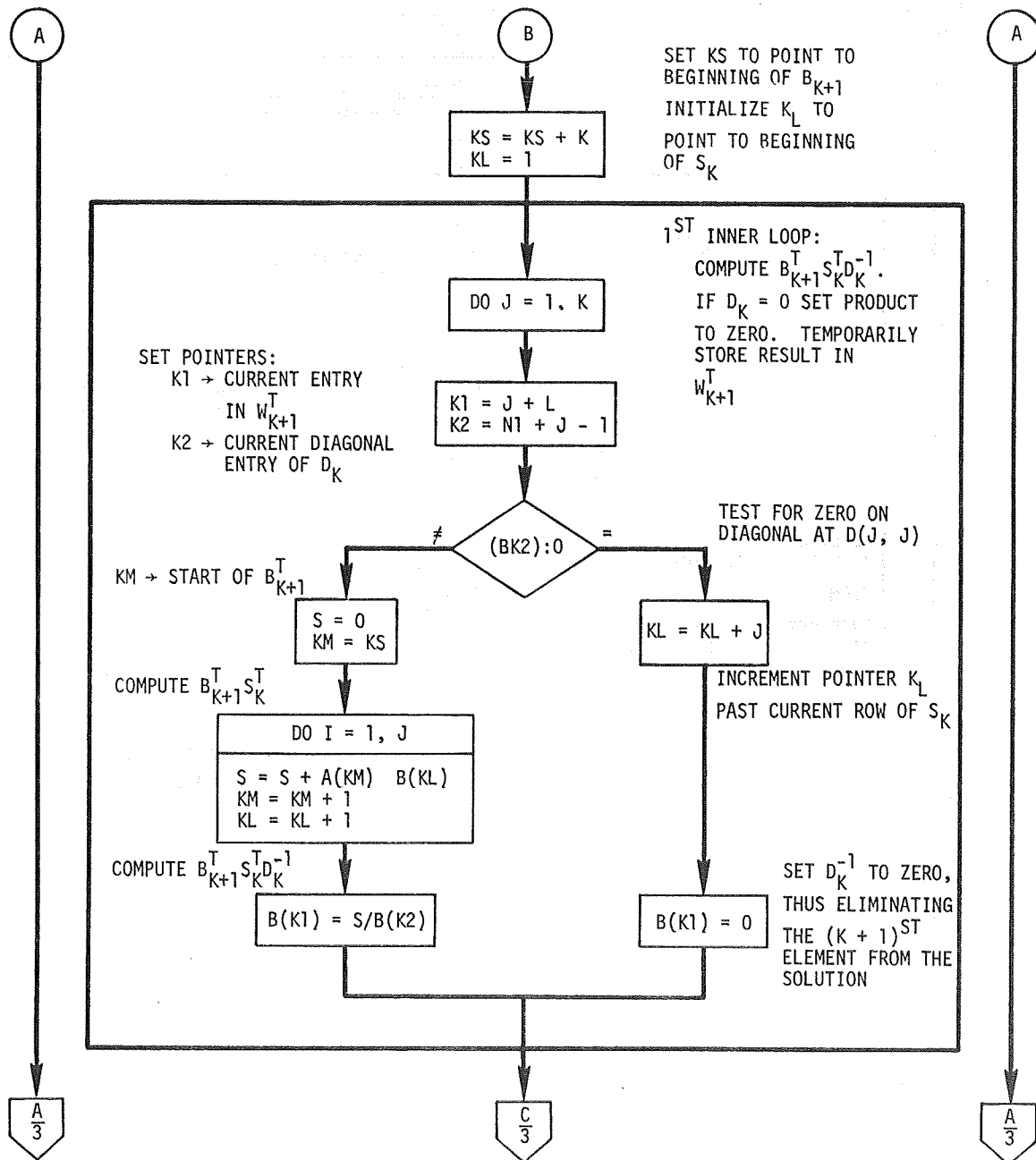


Figure A-19.- Continued.

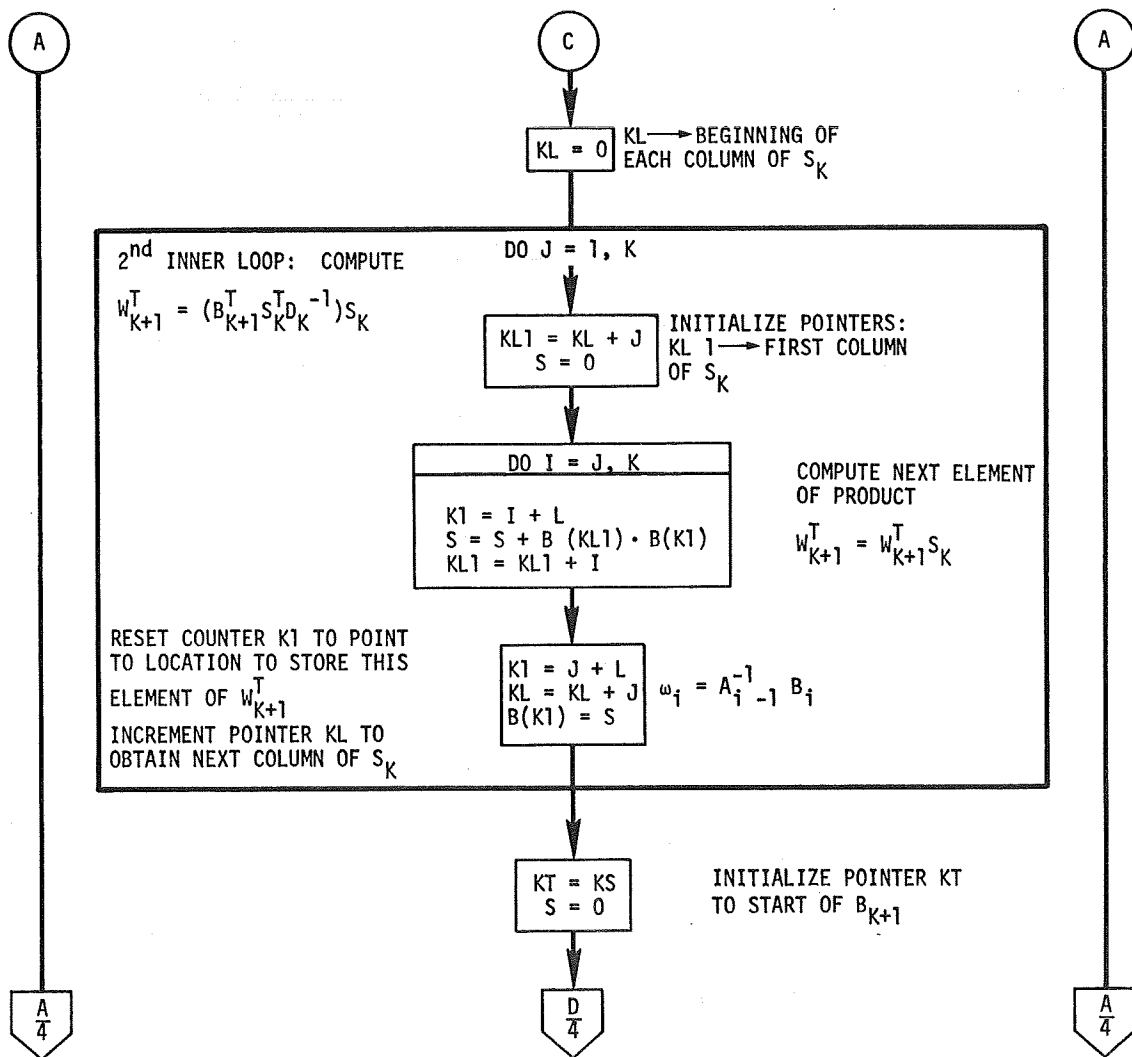


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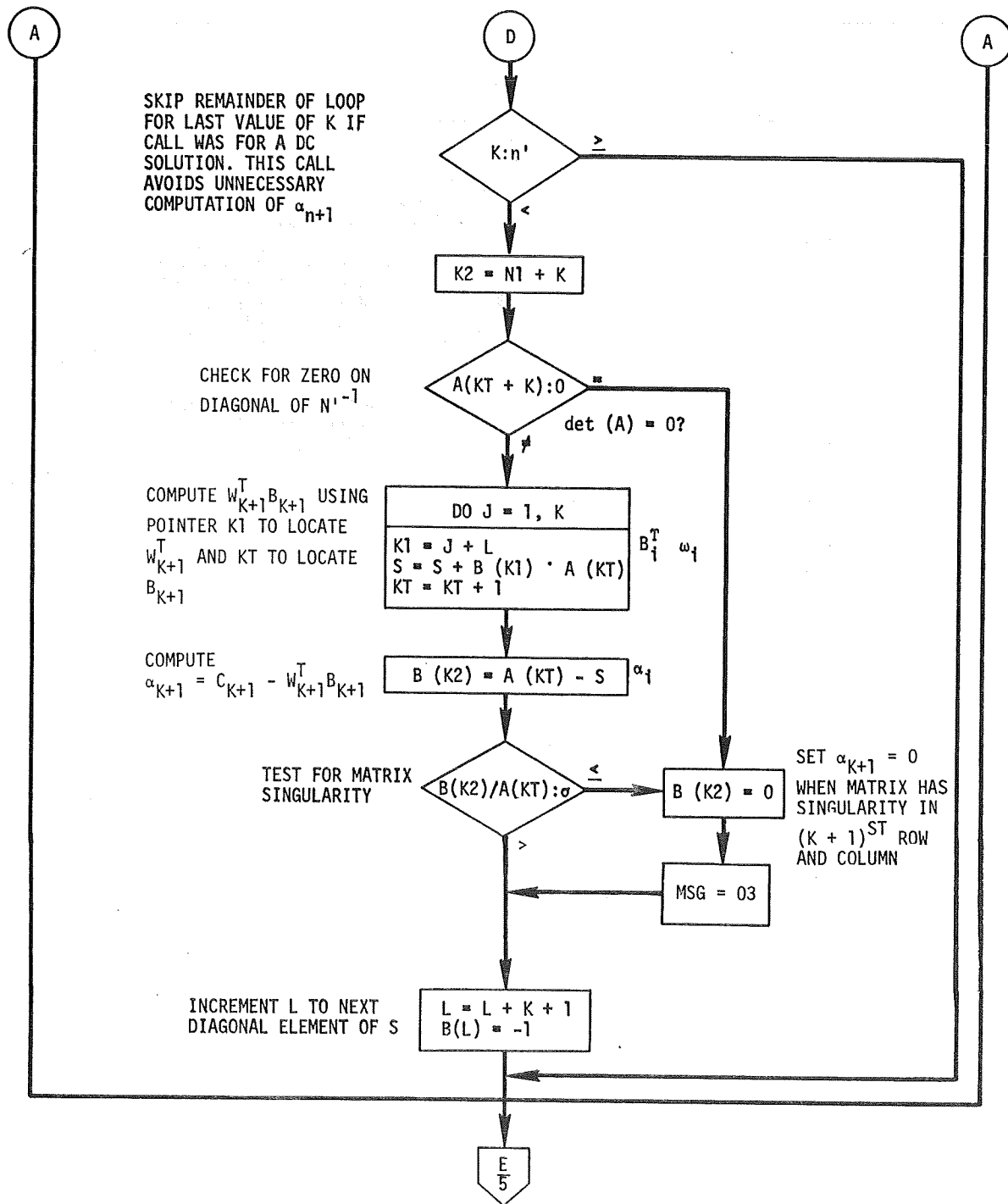


Figure A-19,- Continued.

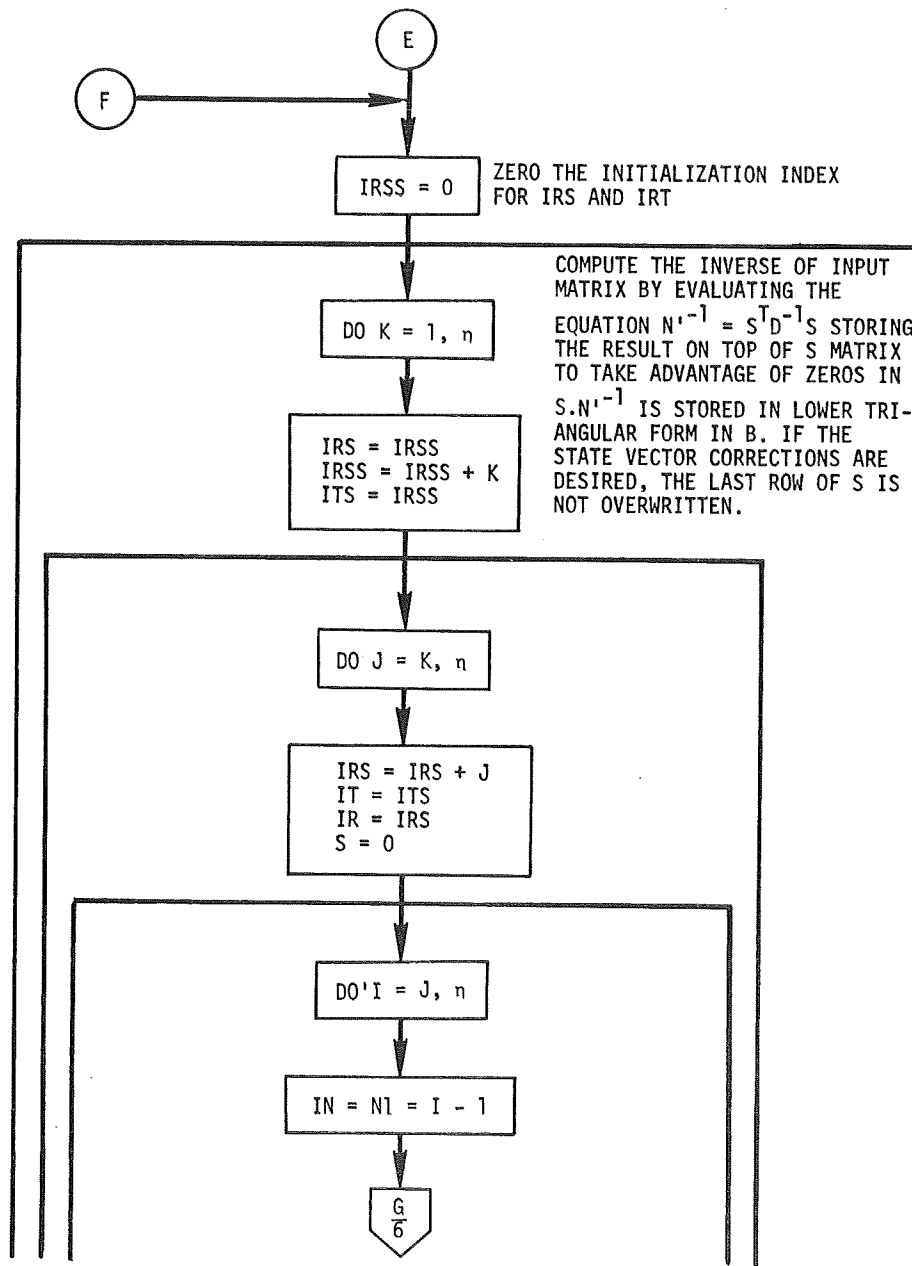


Figure A-19.- Continued.

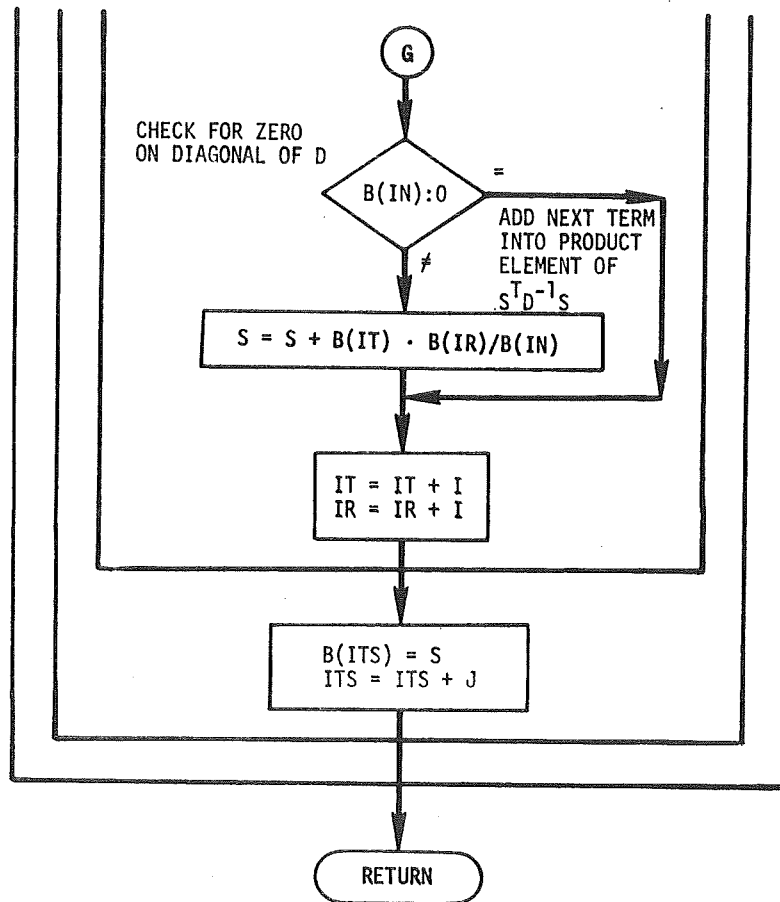


Figure A-19.- Concluded.

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